Power and Transmission Duration Control in Un-Slotted Cognitive Radio Networks

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***Abstract*—We consider an unslotted primary channel with alternating on/off activity and provide a solution to the problem of finding the optimal secondary transmission power and duration given some sensing outcome. The goal is to maximize a weighted sum of the primary and secondary throughput where the weight is determined by the degree of protection and the minimum rate required by the primary terminals. The primary transmitter sends at a fixed power and a fixed rate. Its on/off durations follow an exponential distribution. Two sensing schemes are considered: perfect sensing in which the actual state of the primary channel is revealed, and soft sensing in which the secondary transmission power and time are determined based on the sensing metric directly. We use an upperbound for the secondary throughput assuming that the secondary receiver tracks the instantaneous secondary channel state information. The objective function is non-convex and, hence, the optimal solution is obtained via exhaustive search. Our results show that an increase in the overall weighted throughput can be obtained by allowing the secondary to transmit even when the channel is found to be busy. For the examined system parameter values, the throughput gain from soft sensing is marginal. Further investigation is needed for assessing the potential of soft sensing.**[1]

I. Introduction

Cognitive radio technology is a potential solution to the problem of under-utilization caused by static spectrum allocation. In cognitive radio networks, two classes of users coexist. The primary users are the classical licensed users, whereas the cognitive users, also known as the secondary or unlicensed users, attempt to utilize the resources unused by the primary users following schemes and protocols designed to protect the primary network from interference and service disruption. There are two main scenarios for the primary-secondary coexistence. The first is the overlay scenario where the secondary transmitter checks for primary activity before transmitting. The secondary user utilizes a certain resource, such as a frequency channel, only when it is unused by the primary network. The second scenario is the underlay system where simultaneous transmission is allowed to occur so long as the interference caused by secondary transmission on the primary receiving terminals is limited below a certain level determined by the required primary quality of service.

There is a significant amount of research that pertains to the determination of the optimal secondary transmission parameters to meet certain objectives and constraints. The research in this area has two main flavors. The first takes a physical layer perspective and focuses on the secondary power control problem given the channel gains between the primary and secondary transmitters and receivers. In [1], for instance, the focus is on maximizing a weighted sum rate of secondary users with constraints on the maximum secondary transmitted powers and the maximum tolerable interference level at primary terminals. The traffic pattern on the primary channel is typically not included in this approach save for a primary activity factor such as in [2] and [3].

The second line of research concentrates on primary traffic and seeks to obtain the optimal time between secondary sensing activities in an unslotted system, or the optimal decision, whether to sense or transmit, in a slotted system. Usually under this approach the physical layer is abstracted and the assumption is made that any two packets transmitted in the same time/frequency slot are incorrectly received (e.g., [4], [5], and [6]).

In this paper, we combine aspects of both the overlay and underlay schemes. As in the overlay systems, the secondary transmitter carries out sensing to detect primary activity. However, we allow for secondary transmission **even** when the channel is perfectly sensed to be busy. The rationale behind this is clear from the extreme case of having a very small channel gain between the secondary transmitter and primary receiver enabling the transmitter to work at maximum power without hurting the primary link. Our objective is to find the optimal power and transmission time in order to maximize a weighted sum of primary and secondary rates. The weight used is specified according to the minimum guaranteed rate and the degree of protection needed by the primary link. Though in actual systems, the primary network would have top priority (reflected in a weight close to unity in our formulation detailed below), we present the general case to account for other possible operation scenarios involving networks with no clear priority structure. We consider two sensing schemes: (a) perfect sensing and (b) soft sensing, introduced in [2], where secondary transmission parameters are determined directly from some sensing metric.

The difference between our work and [3] is that in the latter, although the secondary is allowed to transmit even if the primary channel is busy, there is no optimization of the transmission or inter-sensing time because the authors assume that the primary network follows a slotted manner of operation. Also, the notion of soft sensing is not investigated. In [7], sensing is carried out periodically and the secondary transmitter remains silent if the channel is sensed to be busy. In [8], only the transmission time is optimized.

In this paper we make the following contributions. We obtain the optimal sensing-dependent power and transmission time for operation with an unslotted primary network. If the channel is sensed to be free, a certain transmit power is used and the channel is re-sensed after a specific time. A possibly different power and transmission time are used if the channel is busy. Optimizing the transmit power and transmission periods makes use of primary traffic parameters in addition to the physical channels between the transmitters and the receivers. We extend the power and transmission duration control to

[1]This work was supported in part by a grant from the Egyptian National Telecommunications Regulatory Authority

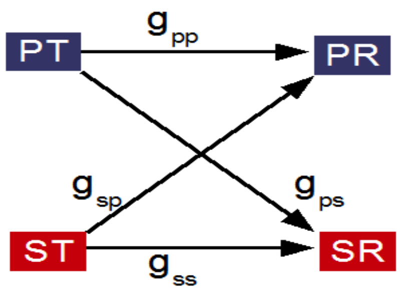

Fig. 1. System model where PT denotes the primary transmitter, PR: primary receiver, SR: secondary receiver and ST: secondary transmitter.

the soft sensing case where the sensing metric is directly used to determine the transmission parameters.

The paper is organized as follows: in section II the system model is introduced. The optimization problem of maximizing the weighted sum rates is provided in Section III. In Section IV, we provide simulation results. Section V concludes the paper.

II. System Model

We consider an unslotted primary channel with alternating on/off primary activity similar to the model employed in [4]. We assume that the probability density function (pdf) of the duration of the on period is exponential and is given by:

$$f_{\rm on}(t) \;\;=\;\; \lambda_{\rm on} \exp\left(-\lambda_{\rm on} t\right),\; t \geq 0 \tag{1}$$

where $\lambda_{\rm on}$ is the reciprocal of the mean on duration $T_{\rm on}$. Similarly, the pdf of the off duration is:

$$f_{\rm off}(t) \;\;=\;\; \lambda_{\rm off} \exp\left(-\lambda_{\rm off} t\right),\; t \geq 0 \tag{2}$$

and $\lambda_{\rm off} = 1/T_{\rm off}$, where $T_{\rm off}$ is the mean of the off duration. The channel utilization factor u is given by

$$u \;\;=\;\; \frac{T_{\rm on}}{T_{\rm on}+T_{\rm off}} \tag{3}$$

Based on results from renewal theory [10], the transition probability that the primary channel is free at time $t^{'}+t$ given that it is free at time $t^{'}$, is given by:

$$P^{00}(t) \;\;=\;\; (1-u) + u \exp\left(-\left[\lambda_{\rm on}+\lambda_{\rm off}\right] t\right) \tag{4}$$

Given that the channel is busy at time $t^{'}$, the transition probability of being free at $t^{'}+t$, is given by:

$$P^{10}(t) \;\;=\;\; (1-u) - (1-u) \exp\left(-\left[\lambda_{\rm on}+\lambda_{\rm off}\right] t\right) \tag{5}$$

The traffic parameters of the primary network can be learned by probing the channel for a specified learning period without transmission. The sensing outcome can be used to estimate the unknown parameters. In the case of perfect sensing, a maximum likelihood estimator can be employed [4]. The parameters $\lambda_{\rm on}$ and $\lambda_{\rm off}$ are obtained via maximizing the likelihood function

$$f\left(S_1, S_2, S_3, ...S_{\rm L} \middle|\, \lambda_{\rm on}, \lambda_{\rm off}\right) \tag{6}$$

where L is the number of sensing outcomes obtained during the learning phase, and S_i is the ith sensing outcome which has one of two values: $S_i = 0$ if the channel is sensed to be free, and $S_i = 1$ for a busy sensing outcome. Using the Markovian property, the likelihood function (6) can be written as

$$f(S_1)f(S_2|S_1)f(S_3|S_2)...f(S_{\rm L}|S_{\rm L-1}) \tag{7}$$

where $f(S_i = v|S_{i-1} = w)$ is the transition probability $P^{wv}\left(\tau_L\right)$ defined above with $v \in \{0,1\}$, $w \in \{0,1\}$, and τ_L is the time between two sensing events. In the simulation section, we present a curve showing the impact of using the learned rather than the true primary traffic parameters. It is important to mention that parameter learning is not the main focus of this work.

The primary transmitter sends with a fixed power $P_{\rm p}$ and at a fixed rate $r_\circ$. A secondary pair tries to communicate over the same channel utilized by the primary terminals. As seen in Figure 1, we denote the gain between primary transmitter and primary receiver as $g_{\rm pp}$, the gain between secondary transmitter and secondary receiver as $g_{\rm ss}$, the gain between primary transmitter and secondary receiver as $g_{\rm ps}$, and finally the gain between secondary transmitter and primary receiver as $g_{\rm sp}$. We assume Rayleigh fading channels and, hence, the channel gains are exponentially distributed with mean values: $\overline{g}_{\rm sp}$, $\overline{g}_{\rm ss}$, $\overline{g}_{\rm ps}$ and $\overline{g}_{\rm pp}$. The channel gains are independent of one another, and the primary and secondary receivers are assumed to know their instantaneous values. In practice, the channels need to be estimated. This can be done through conventional channel training methods, or via exploiting channel reciprocity in systems operating in time-division duplex (TDD) mode. More sophisticated techniques are required by the secondary user to estimate the primary link channel state information utilizing the widely used automatic repeat request (ARQ) feedback from the primary receiver to the primary transmitter [11] and [12], or through cooperation between secondary nodes that could be present close enough to the primary receiver [13].

The secondary transmitter does not transmit while sensing the channel. It senses the channel for a constant time $t_{\rm s}$ assumed to be much smaller than transmission times $T_{\rm on}$ and $T_{\rm off}$. This assumption guarantees that the primary is highly unlikely to change state during the sensing period. Based on the sensing outcome, the secondary transmitter determines its own transmit power and the duration of transmission after which it has to sense the primary channel again.

III. Optimal power level and transmission time

In this section, we explain the problem of finding the optimal secondary transmission time and power given the outcome of the sensing process.

A. Problem Formulation

We formulate the cognitive power and transmission time control problem as an optimization problem with the objective of maximizing a weighted sum of the primary, $R_{\rm p}$, and secondary, $R_{\rm s}$, rates. Specifically, we seek to maximize $\mathbb{E}\{(1-\alpha)\, R_{\rm s} + \alpha R_{\rm p}\}$, where $\mathbb{E}\{.\}$ denotes the expectation operation over the sensing outcome and primary activity. The constant $\alpha \in [0,1]$ is chosen on the basis of the required primary throughput. In order to protect the primary user from interference and service interruption, parameter α should be close to one. In the sequel, however, we study the full range of α so that our results account for other cases where there is no clear priority among the users. The constraints of the optimization problem are that the secondary power lies in the interval $[0, P_{\rm max}]$, and that the time between sensing operations exceeds $t_{\rm s}$. The problem is generally non-convex and, consequently, we resort to exhaustive search to obtain the solution when the number of optimization parameters is small.

In this paper, we consider two sensing scenarios: 1) perfect sensing, and 2) soft sensing where the cognitive transmitter uses some sensing metric γ, say the output of an energy detector, to determine its transmission parameters. Under the soft sensing mode of operation, the range of values of γ is divided into intervals and the transmission power and time are determined based on the interval on which the actual sensing metric γ lies. The parameters to optimize the rate objective function are the transmission powers and times corresponding to each interval and also the boundaries between intervals.

We assume that the primary link is in outage whenever the primary rate $r_\circ$ exceeds the capacity of the primary channel. The primary outage probability when the secondary transmitter emits power p is given by:

$$P_\circ(p) = \Pr\left\{ r_\circ > \log\left(1 + \frac{P_{\rm p}\, g_{\rm pp}}{p\, g_{\rm sp} + \sigma_{\rm p}^2}\right)\right\} \quad (8)$$

where $\sigma_{\rm p}^2$ is the noise variance of the primary receiver. The expression of $P_\circ(p)$ for Rayleigh fading channels is given in the Appendix. We assume that the channel gains vary slowly over time and are almost constant over several epochs of primary and secondary transmission.

For the secondary rate, we assume that the secondary receiver tracks the instantaneous capacity of the channel and, hence, the maximum achievable rate is obtained by averaging over the channel gains and interference levels [7, equation 8]. The ergodic capacity of the secondary channel when the cognitive transmitter emits power p and the primary transmitter is off is expressed as

$$C_\circ(p) = \mathbb{E}_{g_{\rm ss}}\left\{ \log\left(1 + \frac{p\, g_{\rm ss}}{\sigma_{\rm s}^2}\right)\right\} \quad (9)$$

where $\sigma_{\rm s}^2$ is the noise variance of the secondary receiver. When there is simultaneous primary and secondary transmissions, the ergodic capacity of the secondary channel becomes

$$C_1(p) = \mathbb{E}_{g_{\rm ss}, g_{\rm ps}}\left\{ \log\left(1 + \frac{p\, g_{\rm ss}}{P_{\rm p} g_{\rm ps} + \sigma_{\rm s}^2}\right)\right\} \quad (10)$$

We provide expressions for $C_\circ(p)$ and $C_1(p)$ in the Appendix.

B. Perfect Sensing

We mean by perfect sensing that the state of the channel, whether vacant or occupied, is known without error after the channel is sensed. The four parameters used to maximize the weighted sum throughput are $P_{\rm F}$ and $T_{\rm F}$ defined as the power and transmission time when the primary channel is free, and $P_{\rm B}$ and $T_{\rm B}$ corresponding to the busy primary state. Before formulating the optimization problem under perfect sensing, we need to introduce several parameters that pertain to the primary traffic. The probability, π_m, that the mth observation of the channel occurs when the channel is free can be calculated using Markovian property of the traffic model.

$$\pi_m = \pi_{m-1} P^{00}(t_{\rm s} + T_{\rm F}) + (1 - \pi_{m-1}) P^{10}(t_{\rm s} + T_{\rm B}) \quad (11)$$

Another parameter is $P^{\rm ss}$ which is the steady state fraction of time the channel is free when sensed according to some scheme. In the perfect sensing scheme, the channel, when sensed free, is sensed again after $t_{\rm s} + T_{\rm F}$. When sensed busy, it is sensed again after $t_{\rm s} + T_{\rm B}$. Parameter $P^{\rm ss}$ can be obtained by setting $\pi_m = \pi_{m-1} = P^{\rm ss}$ in (11) to get

$$P^{\rm ss} = \frac{P^{10}(t_{\rm s} + T_{\rm B})}{1 - P^{00}(t_{\rm s} + T_{\rm F}) + P^{10}(t_{\rm s} + T_{\rm B})} \quad (12)$$

The average time between sensing times is given by:

$$\mu = P^{\rm ss}(t_{\rm s} + T_{\rm F}) + (1 - P^{\rm ss})(t_{\rm s} + T_{\rm B}) \quad (13)$$

Finally, we also need the average time the channel is free during a period of t units of time if sensed to be free. We denote this quantity by $\delta^\circ(t)$ and is given by [6]

$$\delta^\circ(t) = t - u\left(t + \frac{\exp\left[-(\lambda_{\rm on} + \lambda_{\rm off})\, t\right] - 1}{\lambda_{\rm on} + \lambda_{\rm off}}\right) \quad (14)$$

On the other hand, if the channel is sensed to be busy, the average time the channel is free during a period of t units of time is given by [6]

$$\delta^1(t) = (1 - u)\left(t + \frac{\exp\left[-(\lambda_{\rm on} + \lambda_{\rm off})\, t\right] - 1}{\lambda_{\rm on} + \lambda_{\rm off}}\right) \quad (15)$$

The secondary throughput averaged over primary activity is given by

$$\begin{aligned} \overline{R}_{\rm s} = \; & P^{\rm ss}\frac{\delta^\circ(T_{\rm F})}{\mu} C_\circ(P_{\rm F}) + \\ & P^{\rm ss}\frac{T_{\rm F} - \delta^\circ(T_{\rm F})}{\mu} C_1(P_{\rm F}) + \\ & (1 - P^{\rm ss})\frac{\delta^1(T_{\rm B})}{\mu} C_\circ(P_{\rm B}) + \\ & (1 - P^{\rm ss})\frac{T_{\rm B} - \delta^1(T_{\rm B})}{\mu} C_1(P_{\rm B}) \end{aligned} \quad (16)$$

The first two terms in the above expression are the secondary throughput obtained if the primary is inactive when the channel is sensed. When the sensing outcome is that the channel is free, the secondary emits power $P_{\rm F}$ for a duration $T_{\rm F}$. During the secondary transmission period, the primary transmitter may resume activity. The average amount of time the primary remains idle during a period of length $T_{\rm F}$ after the channel is sensed to be free is obtained by using $t = T_{\rm F}$ in (14). This is the duration of secondary transmission free from interference from the primary transmitter. On the other hand, the primary transmits during secondary operation for an average period of $T_{\rm F} - \delta^\circ(T_{\rm F})$. The last two terms in (16) are the same as the first two but when the channel is sensed to be busy. In this case, the transmit secondary power is $P_{\rm B}$ and the transmission time is $T_{\rm B}$, of which a duration of $\delta^1(T_{\rm B})$ is free, on average, from primary interference.

The primary throughput is given by

$$\begin{aligned} \overline{R}_{\rm p} = \; & r_\circ P^{\rm ss}\frac{T_{\rm F} - \delta^\circ(T_{\rm F})}{\mu}\left[1 - P_\circ(P_{\rm F})\right] + \\ & r_\circ (1 - P^{\rm ss})\frac{T_{\rm B} - \delta^1(T_{\rm B})}{\mu}\left[1 - P_\circ(P_{\rm B})\right] \end{aligned} \quad (17)$$

We ignore the primary throughput that may be achieved during the sensing period because $t_{\rm s}$ is assumed to be much smaller than $T_{\rm on}$and $T_{\rm off}$. The two terms of (17) correspond to the sensing outcomes of the channel being free and busy, respectively. The optimization problem can then be written as

Find: $T_{\rm F}, T_{\rm B}, P_{\rm F}$ and $P_{\rm B}$
That maximize: $(1 - \alpha)\; \overline{R}_{\rm s}(T_{\rm F}, T_{\rm B}, P_{\rm F}, P_{\rm B})$
$+\alpha\; \overline{R}_{\rm p}(T_{\rm F}, T_{\rm B}, P_{\rm F}, P_{\rm B})$
Subject to: $T_{\rm F} \geq 0,\; T_{\rm B} \geq 0,$
$0 \leq P_{\rm F} \leq P_{\max}$ and $0 \leq P_{\rm B} \leq P_{\max}$

C. Soft Sensing

Soft sensing means that the sensing metric is used directly to determine the secondary transmission power and duration. In the sequel, we re-formulate the weighted sum throughput optimization problem assuming quantized soft sensing, where the sensing metric, from a matched filter or an energy detector for instance, is quantized before determining the power and duration of transmission. Let γ be the sensing metric with the known conditional pdfs: $f_\circ(\gamma)$ given that the primary is in the idle state and $f_1(\gamma)$ conditioned on the primary transmitter being active. We assume that the number of quantization levels is $S + 1$. The kth level extends from threshold $\gamma_{k-1}^{\rm th}$ to $\gamma_k^{\rm th}$ assuming that $\gamma_0^{\rm th} = 0$ and $\gamma_{S+1}^{\rm th} = \infty$. The probability that the metric γ is between $\gamma_{k-1}^{\mathrm{th}}$ and γ_k^{th} when the primary channel is free is given by

$$\begin{aligned}\epsilon_k &= \Pr\{\gamma_{k-1}^{\mathrm{th}} \leq \gamma \leq \gamma_k^{\mathrm{th}} | \text{channel is free}\} \\ &= \int_{\gamma_{k-1}^{\mathrm{th}}}^{\gamma_k^{\mathrm{th}}} f_0(\gamma)\, d\gamma \qquad (18)\end{aligned}$$

where $k = 1, 2, \cdots (S+1)$. On the other hand, The probability that γ is between $\gamma_{k-1}^{\mathrm{th}}$ and γ_k^{th} when the primary channel is busy is given by

$$\begin{aligned}\vartheta_k &= \Pr\{\gamma_{k-1}^{\mathrm{th}} \leq \gamma \leq \gamma_k^{\mathrm{th}} | \text{channel is busy}\} \\ &= \int_{\gamma_{k-1}^{\mathrm{th}}}^{\gamma_k^{\mathrm{th}}} f_1(\gamma)\, d\gamma \qquad (19)\end{aligned}$$

When γ is between $\gamma_{k-1}^{\mathrm{th}}$ and γ_k^{th}, the secondary transmitted power is P_k and the duration of transmission is T_k.

As in the perfect sensing case, the probability that mth observation of the channel happens when the channel is free, denoted by π_{m}, can be calculated using Markovian property of the channel model.

$$\begin{aligned}\pi_m &= \pi_{m-1} \sum_{k=1}^{S+1} \epsilon_k P^{00}(t_{\mathrm{s}} + T_k) \\ &+ (1 - \pi_{m-1}) \sum_{k=1}^{S+1} \vartheta_k P^{10}(t_{\mathrm{s}} + T_k) \qquad (20)\end{aligned}$$

At steady state, $\pi_{m-1} = \pi_m$ and the steady state probability of sensing the channel while it is free becomes

$$P^{\mathrm{ss}} = \frac{\sum_{k=1}^{S+1} \vartheta_k P^{10}(t_{\mathrm{s}} + T_k)}{1 - \sum_{k=1}^{S+1} \epsilon_k P^{00}(t_{\mathrm{s}} + T_k) + \sum_{k=1}^{S+1} \vartheta_k P^{10}(t_{\mathrm{s}} + T_k)} \qquad (21)$$

The average time between sensing events is given by

$$\mu = P^{\mathrm{ss}} \sum_{k=1}^{S+1} \epsilon_k \left(t_{\mathrm{s}} + T_k\right) + (1 - P^{\mathrm{ss}}) \sum_{k=1}^{S+1} \vartheta_k \left(t_{\mathrm{s}} + T_k\right) \qquad (22)$$

The mean secondary throughput averaged over the primary activity and the sensing metric is given by

$$\begin{aligned}\overline{R}_{\mathrm{s}} &= P^{\mathrm{ss}} \sum_{k=1}^{S+1} \epsilon_k \left[\frac{\delta^\circ(T_k)}{\mu} C_\circ\left(P_k\right) + \frac{T_k - \delta^\circ(T_k)}{\mu} C_1\left(P_k\right)\right] \\ &+ (1 - P^{\mathrm{ss}}) \sum_{k=1}^{S+1} \vartheta_k \left[\frac{\delta^1(T_k)}{\mu} C_\circ\left(P_k\right) \right. \\ &\left. + \frac{T_k - \delta^1(T_k)}{\mu} C_1\left(P_k\right)\right] \qquad (23)\end{aligned}$$

The mean primary throughput is

$$\begin{aligned}\overline{R}_{\mathrm{p}} &= r_\circ P^{\mathrm{ss}} \sum_{k=1}^{S+1} \epsilon_k \frac{T_k - \delta^\circ(T_k)}{\mu} \left[1 - P_\circ\left(P_k\right)\right] + \\ & r_o(1 - P^{ss}) \sum_{k=1}^{S+1} \vartheta_k \frac{T_k - \delta^1(T_k)}{\mu} \left[1 - P_\circ\left(P_k\right)\right] \qquad (24)\end{aligned}$$

IV. Numerical Results

In this section we present simulation results for the perfect and soft sensing schemes discussed in Section III. The weighted sum rate maximization problem is non-convex, hence, we do exhaustive search to obtain the optimal parameters. The parameters used in our simulations presented here are: $T_{\mathrm{on}} = 4$, $T_{\mathrm{off}} = 5$, $t_{\mathrm{s}} = 0.05$, $r_\circ = 4.5$ nats, $\sigma_{\mathrm{s}}^2 = \sigma_{\mathrm{p}}^2 = 1$, $P_{\mathrm{p}} = 100$, $P_{\max} = 10$, $\overline{g}_{\mathrm{ss}} = 2$, $\overline{g}_{\mathrm{pp}} = 3$, and $\overline{g}_{\mathrm{ps}} = .03$. In order to do the exhaustive search, we have imposed an artificial upperbound on transmission time equal to 20. We analyze the results for perfect sensing in Subsection IV-A and for soft sensing in Subsection IV-B. The parameters for channels **A** and **B** used in the analysis are the same except for $\overline{g}_{\mathrm{sp}}$ which is equal to 2 for channel **A** and 0.2 for channel **B**.

A. Perfect Sensing

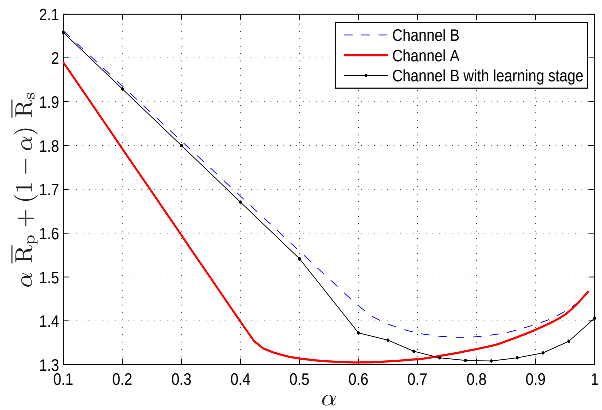

Fig. 2. Perfect sensing weighted sum throughput versus α for channels **A** and **B**.

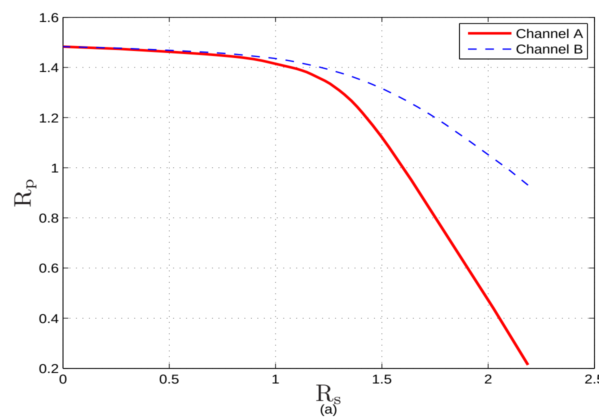

Fig. 3. Rate region, $\mathrm{R_s}$ vs. $\mathrm{R_p}$ for channels **A** and **B**.

The weighted sum throughput versus α is shown in Figure 2 for channels **A** and **B**, whereas the rate region depicting the variation of secondary with primary throughput is provided in Figure 3. It is clear from Figure 2 that as the gain $\overline{g}_{\mathrm{sp}}$ increases, the level of interference at the primary receiver increases leading to lower data rates. We also include here the curve for the mean weighted sum throughput for channel **B** when the traffic parameters λ_{on} and λ_{off} are estimated. The learning parameters (explained in Section II) are $L = 25$ and $\tau_{\mathrm{L}} = 0.5$. It is clear from the figure that there is a degradation in weighted sum throughput due to the uncertainty regarding the traffic parameters. As we have emphasized earlier, learning is not the main focus of this paper, but will be the subject of future investigation.

The optimal transmission power and time parameters for channel **A** are given in Figure 4. For small α value, which corresponds to giving more importance to the secondary throughput, the secondary transmitter emits $P_{\max}$ whether the channel is sensed to be free or busy. The transmission time for both sensing outcomes are the maximum possible. Recall that this maximum is artificial and is

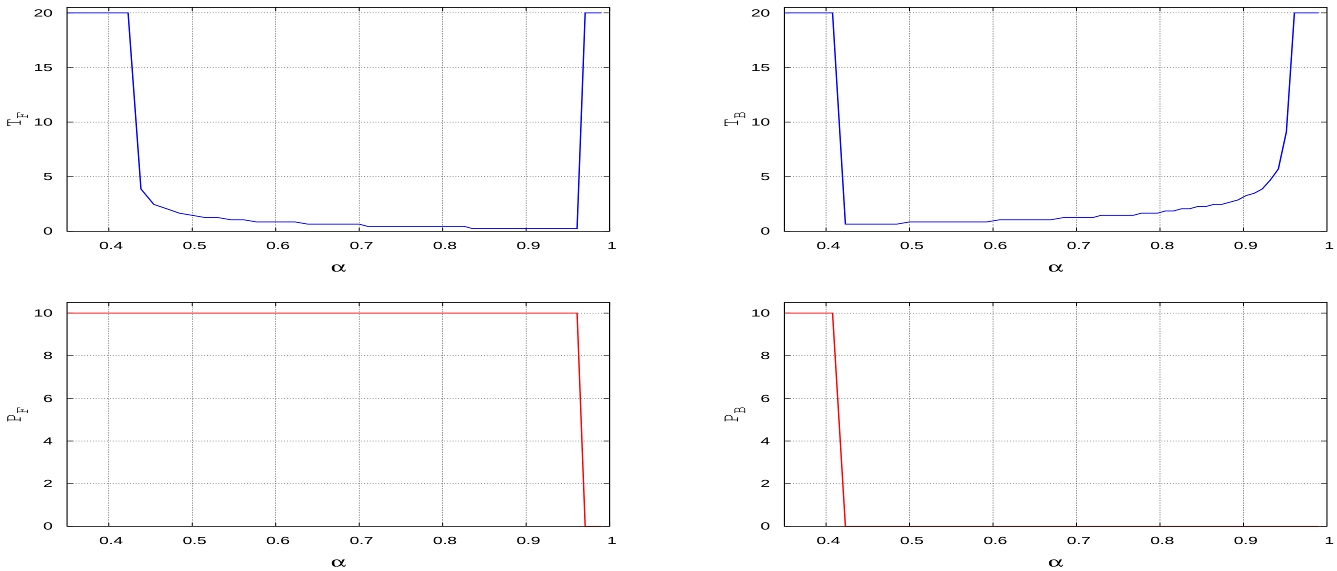

Fig. 4. Perfect sensing power and transmission time results for channel **A**.

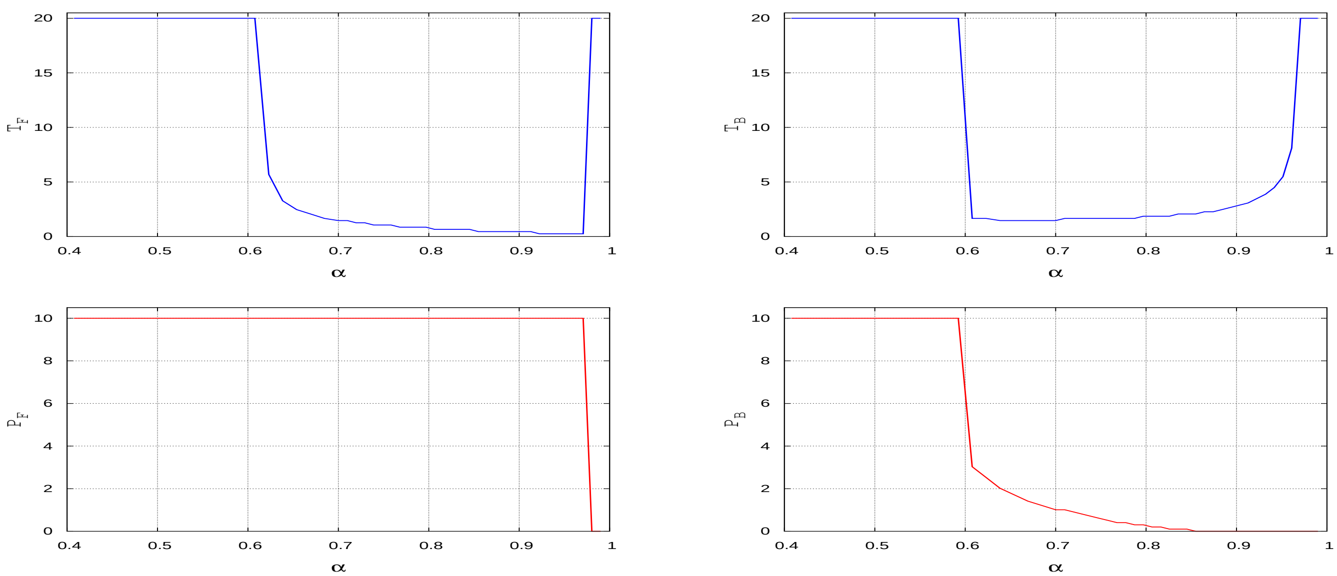

Fig. 5. Perfect sensing power and transmission time results for channel **B**.

imposed by the exhaustive search solution. In fact, for α approaching zero, the secondary transmitter sends with $P_{\max}$ continuously without the need to sense the channel again. If the optimal $P_{\mathrm{F}} = P_{\mathrm{B}}$, then sensing becomes superfluous because the exact same power would be used regardless of the sensing outcome. As α increases, the power transmitted when the channel is sensed to be busy is reduced below $P_{\max}$. In addition, the transmission times are reduced for more frequent checking of primary activity. As α approaches unity, the secondary transmitter is turned off and the channel is not sensed. Figure 5 gives the optimal transmission parameters for channel **B**. It is evident from the figure that as the level of interference from secondary transmitter to primary receiver is decreased, P_{B} becomes lower than $P_{\max}$ at a higher α compared to **A**.

B. Soft Sensing

In the soft sensing case, the optimization parameters are $2\,(S+1)$ transmission powers and times corresponding to each quantization level. There are also S thresholds defining the boundaries of the quantization levels. Hence, the total number of parameters is $3S+2$. The conditional distributions of the sensing metric γ used in the simulations are $f_{\circ}\,(\gamma) = \exp\,(-\gamma)$ and $f_1\,(\gamma) = \exp\,(-\,[\gamma+\gamma_{\circ}])\ \ I_{\circ}\,\left(2\sqrt{\gamma\gamma_{\circ}}\right)$, where $I_{\circ}$ is the zero order modified Bessel function and $\gamma_{\circ}$ is a parameter related to the mean value of $f_1\,(\gamma)$. We present here the results for one and two thresholds. The case of one threshold corresponds to the imperfect sensing case where the primary is assumed to be active when γ exceeds some threshold and inactive otherwise. The false alarm probability is given by ϵ_2, whereas the miss detection probability is ϑ_1. Figure 6 gives the optimal threshold as a function of α and for $\gamma_{\circ} = 3$. As is evident from the figure, the optimal threshold decreases with α. Under the imperfect sensing interpretation of the one threshold case, this means that as α increases putting more emphasis on the primary rate, the required false alarm probability is increased while the miss detection probability is decreased to reduce the chance of collision with the primary user. Figure 7 shows the weighted sum throughput using

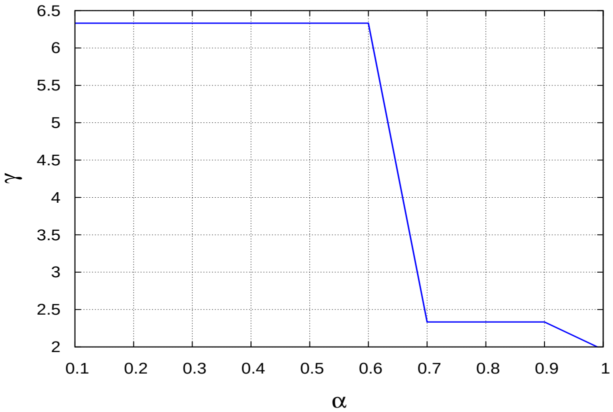

Fig. 6. Soft sensing optimal threshold for channel **B**.

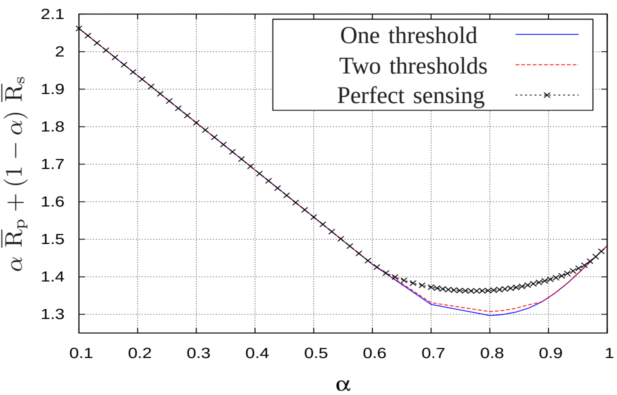

Fig. 7. Soft sensing weighted sum throughput versus α using one and two thresholds for channel **B**and comparison with perfect sensing.

one and two thresholds for channel **B** and $\gamma_\circ = 3$. There is a range of α values for which the two-threshold scheme improves very slightly the weighted sum rates.

V. Conclusion

We have investigated the problem of specifying transmission power and duration in an underlay unslotted cognitive radio network, where the primary transmission duration follows an exponential distribution. We used an upperbound for the secondary throughput, and obtained, numerically, the optimal secondary transmission power and duration that maximize a weighted sum of the primary and secondary throughputs. Our results also showed that an increase in the overall weighted throughput can be obtained by allowing the secondary to transmit even when the channel is found to be busy. We extended our formulation to the soft sensing case where the decision of the secondary transmission power and duration depends on the quantized value of the sensing metric, rather than on the binary decision of whether the channel is free or not. However, our preliminary results show that the gain of using this scheme, and for the range of parameters we have simulated, are minimal.

VI. Appendix

We provide here expressions for (8), (9), and (10). Assuming that g_{pp} and g_{sp} are independent and exponentially distributed with means $\overline{g}_{\mathrm{pp}}$ and $\overline{g}_{\mathrm{sp}}$, the outage probability (8) can be written as

$$\begin{aligned} P_\circ(p) &= \Pr\left\{ r_\circ > \log\left(1 + \frac{a g_{\mathrm{pp}}}{b g_{\mathrm{sp}} + 1}\right)\right\} \\ &= 1 - \frac{P_{\mathrm{p}}\overline{g}_{\mathrm{pp}}}{P_{\mathrm{p}}\overline{g}_{\mathrm{pp}} + pc\overline{g}_{\mathrm{sp}}} \exp\left(-\frac{c\sigma_{\mathrm{p}}^2}{P_{\mathrm{p}}\overline{g}_{\mathrm{pp}}}\right) \end{aligned} \tag{25}$$

where $a = P_{\mathrm{p}}/\sigma_p^2$, $b = p/\sigma_s^2$ and $c = \exp(r_\circ) - 1$. Given an exponential distribution for g_{ss} with mean $\overline{g}_{\mathrm{ss}}$, (9) becomes

$$C_\circ(p) = \int_0^\infty \log\left(1 + \frac{p g_{\mathrm{ss}}}{\sigma_s^2}\right) \frac{1}{\overline{g}_{\mathrm{ss}}} \exp\left(-\frac{g_{\mathrm{ss}}}{\overline{g}_{\mathrm{ss}}}\right) dg_{\mathrm{ss}} \tag{26}$$

Defining $\Psi(x) = \int_x^\infty \exp(-\mu)/\mu \, d\mu$, it is straightforward to show that

$$C_\circ(p) = \exp\left(\frac{\sigma_s^2}{p\overline{g}_{\mathrm{ss}}}\right) \Psi\left(\frac{\sigma_s^2}{p\overline{g}_{\mathrm{ss}}}\right) \tag{27}$$

Assuming that g_{ss} and g_{ps} are independent and have means $\overline{g}_{\mathrm{ss}}$ and $\overline{g}_{\mathrm{ps}}$, respectively, when $p\overline{g}_{\mathrm{ss}} \neq P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}$, (10) can be expressed as

$$\begin{aligned} C_1(p) &= \frac{p\overline{g}_{\mathrm{ss}}}{p\overline{g}_{\mathrm{ss}} - P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}} \Big[\exp\left(\frac{\sigma_s^2}{p\overline{g}_{\mathrm{ss}}}\right) \Psi\left(\frac{\sigma_s^2}{p\overline{g}_{\mathrm{ss}}}\right) - \\ &\qquad \exp\left(\frac{\sigma_s^2}{P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}}\right) \Psi\left(\frac{\sigma_s^2}{P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}}\right) \Big] \end{aligned} \tag{28}$$

In the case $p\overline{g}_{\mathrm{ss}} = P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}$,

$$C_1(p) = 1 - \frac{\sigma_s^2}{p\overline{g}_{\mathrm{ss}}} \exp\left(\frac{\sigma_s^2}{p\overline{g}_{\mathrm{ss}}}\right) \Psi\left(\frac{\sigma_s^2}{p\overline{g}_{\mathrm{ss}}}\right) \tag{29}$$

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