POWER AND TRANSMISSION TIME CONTROL IN COGNITIVE RADIO NETWORKS BASED ON SENSING OUTCOME OR PRIMARY FEEDBACK

A DISSERTATION
SUBMITTED TO THE PROGRAM IN WIRELESS TECHNOLOGY
AND THE COMMITTEE ON GRADUATE STUDIES
OF NILE UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE

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August 2010

To my family
and my best friend “Mohamed Hevzy”

Abstract

We consider an interference channel composed of one primary and one secondary links. Our objective is to obtain the secondary transmission parameters in order to maximize a weighted sum of primary and secondary throughput. We consider the two cases of an unslotted primary operation and slotted operation. In the former case, the primary activity alternates between on and off according to some distributions. Our optimization parameters are the secondary power and transmission time given the sensing outcome. In the latter case, transmission takes place over the fixed slot duration. What we optimize is the power on the basis of the feedback sent by primary receiver to its transmitter. In the unslotted case, between the sensing instants, the secondary receiver does not know when exactly the primary transmitter is active. We drive an upper and lower bounds on the ergodic capacity of the secondary link. The upper bound is derived assuming perfect knowledge of primary activity at the secondary receiver. The lower bound is derived on the basis of the worst uncertainty regarding primary activity. We obtain the maximum gap between the upper and lower bounds as a function of the primary traffic parameters and secondary sensing-dependent transmission times. Then, we use these formulas to obtain the optimal transmission parameters. Two sensing schemes are considered: perfect sensing in which the actual state of the primary channel is revealed, and soft sensing in which the secondary transmission power and time are determined based on the sensing metric directly. We use an upperbound for the secondary throughput assuming that the secondary receiver tracks the instantaneous secondary channel state information. The objective function is non-convex and, hence, the optimal solution is obtained via exhaustive search. Our results show that an increase in the overall

weighted throughput can be obtained by allowing the secondary to transmit even when the channel is found to be busy. For the examined system parameter values, the throughput gain from soft sensing is marginal. Further investigation is needed for assessing the potential of soft sensing. Finally we investigate the slotted scheme. In this scheme, the primary receiver feedback message is used to obtain the optimum secondary user power to maximize the accumulated reward. Different secondary user policies are considered: greedy, genie aided and causal.

Acknowledgments

I would like to thank my Parents for everything. They taught me everything I knew. They supported me every moment in my life.

I want to thank my Advisor Prof. Ahmed Sultan for his technical and personal support, For his guiding and for being beside me in every moment. He was not only my supervisor, but a father and a brother. I will be happy if I could be one day the half the man he is. I will always owe him.

I would like to thank my co-advisor and my program director Prof. Mohamed Nafie for his directions and also I would like to acknowledge Prof. Hesham El Gamal for giving me the chance to join the WINC team.

The unforgettable dear professor, Mohmed Khairy. He was my advisor in undergraduate and serving on my thesis committee. He was always and still the older brother, the friend and the support in most of my decision in my personal and academic life.

I also can not forget the moments I have spent in Nile University with all of my colleagues. Special thanks for my best friends specially Tamer El-Azzony, Toka sabry and Sonia Soliman. Finally, I will never forget my colleagues in WINC specially Muhamed Farhoud, John George. And also my office mates Doha, Marwa and Rania in addition to all the Alex guys for their wonderful support and their friendship. Also, the first year RAs specially Laila Hesham, Ahmed Amr and Ahmed Osama for their friendship.

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Chapter 1

Introduction

As a result for the incremental demand on the wireless communication and its services, the technology of cognitive radio becomes a very important area in the wireless communication research. Cognitive radio network is considered as the potential solution for the under-utilized spectrum problem. For example, in the mobile communication, not all the spectrum or the mobile channels in a certain area and a certain time is used. According to Federal Communications Commission (FCC) [1], temporal and geographical variations in the utilization of the assigned spectrum range from 15% to 85% as shown in Figure [1.1]. That is to say, the classical fixed spectrum assignment results in an underutilized spectrum usage. Consequently, to enhance this utilization dynamic spectrum access can be used.

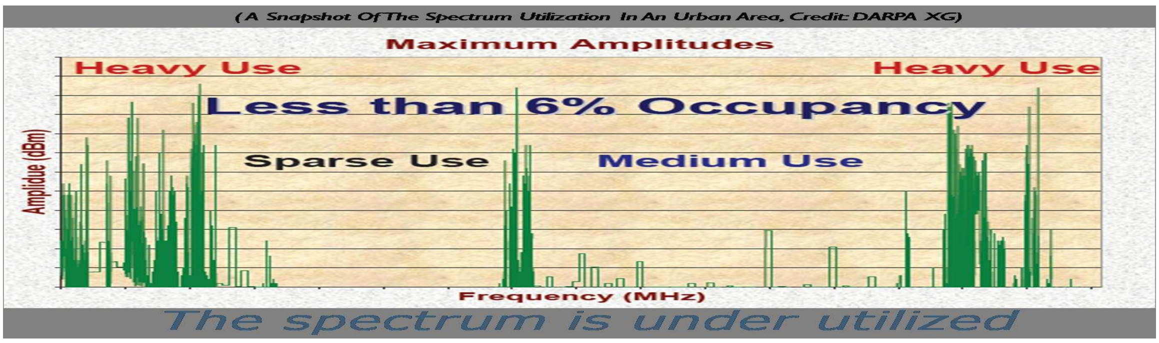

Figure 1.1: Snapshot of the spectrum to show spectrum under-utilization

In cognitive radio networks, two classes of users coexist. The primary users (PUs) are the classical licensed users (A licensed mobile operator for example), whereas the cognitive users, also known as the secondary (SUs) or unlicensed users , attempt to utilize the resources unused by the primary users following schemes and protocols designed to protect the primary network from interference and service disruption. Hence, the secondary user target may be maximize the throughput, mitigate the interference, facilitate inter-operability,... etc. However, SUs have some hard missions to reach their goal. According to the strategy used, the missions could be

- Spectrum sensing.
- Spectrum access and resources control.
- Spectrum evacuation, when primary user returns to the channel.

There are two main scenarios for the primary-secondary coexistence. The first is the overlay scenario where the secondary transmitter checks for primary activity before transmitting. The secondary user utilizes a certain resource, such as a frequency channel, only when it is unused by the primary network i.e., The secondary user will send only when the primary user is not present. The second scenario is the underlay system where simultaneous transmission is allowed to occur so long as the interference caused by secondary transmission on the primary receiving terminals is limited below a certain level determined by the required primary quality of service. Here secondary users are allowed to send **even** if the primary user exist as long as the constraints required by the primary is not violated.

In this thesis, we derive the capacity formula for the secondary user in case of un-slotted cognitive radio. In this model the primary user has an alternating on/off activity and changes his activity randomly. The secondary user chooses the power and transmission time according to the sensing outcome. After the transmission time, the secondary user re-senses the channel again. Next, we obtain the transmission parameters, power and time, for the secondary user to maximize the weighted sum throughput. Finally, we exploit how to use the primary network feedbacks to control the secondary user power.

1.1 Overview of Related Work

There is a significant amount of research that pertains to the determination of the optimal secondary transmission parameters to meet certain objectives and constraints. The research in this area has two main flavors. The first takes a physical layer perspective and focuses on the secondary power control problem given the channel gains between the primary and secondary transmitters and receivers. In [2], for instance, the focus is on maximizing a weighted sum rate of secondary users with constraints on the maximum secondary transmitted powers and the maximum tolerable interference level at primary terminals. The traffic pattern on the primary channel is typically not included in this approach save for a primary activity factor such as in [3] and [4]. The second line of research concentrates on primary traffic and seeks to obtain the optimal time between secondary sensing activities in an unslotted system, or the optimal decision, whether to sense or transmit, in a slotted system. Usually under this approach the physical layer is abstracted and the assumption is made that any two packets transmitted in the same time/frequency slot are incorrectly received (e.g., [5], [6], and [7]).

In this thesis, we combine aspects of both the overlay and underlay schemes. As in the overlay systems, the secondary transmitter carries out sensing to detect primary activity. However, we allow for secondary transmission **even** when the channel is perfectly sensed to be busy. The rationale behind this is clear from the extreme case of having a very small channel gain between the secondary transmitter and primary receiver enabling the transmitter to work at maximum power without hurting the primary link. Our objective is to find the optimal power and transmission time in order to maximize a weighted sum of primary and secondary rates. The weight used is specified according to the minimum guaranteed rate and the degree of protection needed by the primary link. Though in actual systems, the primary network would have top priority (reflected in a weight close to unity in our formulation detailed below), we present the general case to account for other possible operation scenarios involving networks with no clear priority structure. We consider two sensing schemes: (a) perfect sensing and (b) soft sensing, introduced in [3], where secondary transmission parameters are determined directly from some sensing metric. The difference between our work and [4] is that in the latter, although the secondary is allowed to transmit even if the primary channel is busy, there is no optimization of the transmission or inter-sensing time because the authors assume that the primary network follows a slotted manner of operation. Also, the notion of soft sensing is not investigated. In [8], sensing is carried out periodically and the secondary transmitter remains silent if the channel is sensed to be busy. In [9], only the transmission time is optimized.

`However,` to formulate the objective function in our scenario we need to derive the capacity for the secondary user. There has been a significant amount of research on the interference channel with an implicit perfect synchronization assumption. The "asynchronous" interference channel with uncoordinated transmitter-receiver pairs is studied in [10] and [11]. In [12] and [13] the exact interference pattern is assumed to be known. Another type of work which is the arbitrary varying channel which consider no information about the channel state [14]. In this thesis we focus on this model as it is more consistent with the actual operation of coexisting primary and secondary users. Specifically, we assume knowledge of the **(a)** channel gains and **(b)** traffic statistics, but not the exact temporal interference pattern inflicted by the primary transmitter on the secondary receiver. **In literature,** the underlay unslotted cognitive user capacity formulas were provided before without justification or disclaimer. In [24] equation (5), same formulas are used without a proof assuming only genie aided who informs the secondary user with the exact primary activity. However, we proved that this is the upper bound and we characterize exactly the gap when the exact activity is not known. In [25] page 3, the authors used, directly and without justification, the expression for the ergodic capacity when primary is present if a miss detection happens.

Another recent flavor of research on power control is by exploiting the feedback message between the primary receiver and primary transmitter. Hearing the ACKs and NACKs of primary user by secondary user allows the SU to have an indication about the primary channel condition. Also it gives a measure about the interference that can be added to the primary link without affecting the communication. The work done by [19], [20] , [21] and [16] is considered as basics in exploiting feedbacks on secondary user power control. We extend the system model by making the direct primary channel and interference channel from secondary on primary changing their states between good and bad. Most of other works consider only the change of the direct channel. We also investigate the problem formulation, which is very similar to [20], in case of greedy, causal and genie aided secondary user.

1.2 Contribution

In the following, we summarize the contribution of this thesis to the solution of optimizing secondary transmission parameters in the presence of primary user:

- We drive an upper and lower bounds on the ergodic capacity of the secondary link. Where, the cognitive transmitter perfectly senses the channel and transmits with a certain power and for a certain duration depending on the primary channel state, then it re-senses the channel. The primary system is unslotted and the primary activity follows an on/off alternating renewal process. Between the sensing instants, the secondary receiver does not know when exactly the primary transmitter is active.

- We obtain the optimal sensing-dependent power and transmission time for operation with an unslotted primary network. If the channel is sensed to be free, a certain transmit power is used and the channel is re-sensed after a specific time. A possibly different power and transmission time are used if the channel is busy.

- We extend the power and transmission duration control to the soft sensing case where the sensing metric is directly used to determine the transmission parameters.

- We investigate exploiting the feedback message sent by the primary receiver to control secondary user transmission parameter in case of direct primary channel and interference channel from secondary on primary alternates their states between good and bad.

- We also investigate the problem formulation, which is very similar to [20], in case of greedy, causal and genie aided secondary user.

1.3 Thesis Organization

We start by showing our system model in chapter 2. Chapter3 introduces the proof for underlay unslotted cognitive radio user capacity. In chapter 4 the optimization problem is formulated for both perfect sensing and soft sensing cases. Another power control scheme is investigated in chapter 5. Finally, chapter 6 concludes the thesis and propose some future work.

Chapter 2

System Model

We consider an unslotted primary channel with alternating on/off primary activity as shown in Figure 2.1 similar to the model employed in [5]. We assume that the probability density function (pdf) of the duration of the on period is exponential and is given by:

$$f_{\text{on}}(t) \;=\; \lambda_{\text{on}} \exp\left(-\lambda_{\text{on}} t\right), \; t \geq 0 \tag{2.1}$$

Where λ_{on} is the reciprocal of the mean on duration T_{on}. Similarly, the pdf of the off duration is:

$$f_{\text{off}}(t) \;=\; \lambda_{\text{off}} \exp\left(-\lambda_{\text{off}} t\right), \; t \geq 0 \tag{2.2}$$

and $\lambda_{\text{off}} = 1/T_{\text{off}}$, where T_{off} is the mean of the off duration. The channel utilization factor u is given by

$$u \;=\; \frac{T_{\text{on}}}{T_{\text{on}} + T_{\text{off}}} \tag{2.3}$$

Based on results from renewal theory [15], and for for exponentially distributed on/off durations, the transition probability that the primary channel is free at time $t' + t$

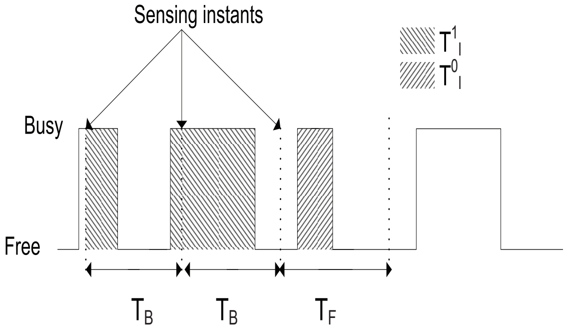

Figure 2.1: System operation in time. Due to the un-slotted nature of primary traffic, the primary transmitter switches activity during secondary transmission period, whether the channel is sensed to be free or busy at the beginning. Parameter T_{I}^{0} denotes the time over which there is simultaneous primary and secondary transmissions given that the channel at the beginning of the transmission cycle is sensed to be free. Parameter T_{I}^{1} is defined similarly assuming that the initial sensing outcome is busy.

given that it is free at time $t^{'}$, is given by:

$$P^{00}(t) \quad = \quad (1-u) + u \exp\left(-\left[\lambda_{\mathrm{on}} + \lambda_{\mathrm{off}}\right] t\right) \tag{2.4}$$

Given that the channel is busy at time $t^{'}$, the transition probability of being free at $t^{'} + t$, is given by:

$$P^{10}(t) \quad = \quad (1-u) - (1-u) \exp\left(-\left[\lambda_{\mathrm{on}} + \lambda_{\mathrm{off}}\right] t\right) \tag{2.5}$$

The primary transmitter sends with a fixed power P_{p} and at a fixed rate $r_{\circ}$. A secondary pair tries to communicate over the same channel utilized by the primary terminals. As seen in Figure 2.2, we denote the gain between primary transmitter and primary receiver as g_{pp}, the gain between secondary transmitter and secondary receiver as g_{ss}, the gain between primary transmitter and secondary receiver as g_{ps}, and finally the gain between secondary transmitter and primary receiver as g_{sp}.

We assume Rayleigh fading channels and, hence, the channel gains are exponentially distributed with mean values: $\overline{g}_{\mathrm{sp}}$, $\overline{g}_{\mathrm{ss}}$, $\overline{g}_{\mathrm{ps}}$ and $\overline{g}_{\mathrm{pp}}$. The channel gains are independent of one another, and the primary and secondary receivers are assumed

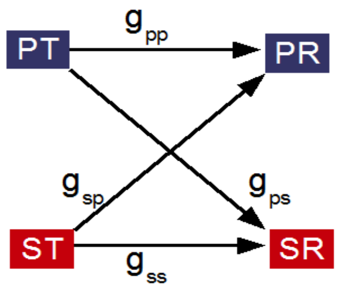

Figure 2.2: System model where PT denotes the primary transmitter, PR: primary receiver, SR: secondary receiver and ST: secondary transmitter.

to know their instantaneous values. In practice, the channels need to be estimated. This can be done through conventional channel training methods, or via exploiting channel reciprocity in systems operating in time-division duplex (TDD) mode. More sophisticated techniques are required by the secondary user to estimate the primary link channel state information utilizing the widely used automatic repeat request (ARQ) feedback from the primary receiver to the primary transmitter [16] and [17], or through cooperation between secondary nodes that could be present close enough to the primary receiver [18].

The secondary transmitter does not transmit while sensing the channel. It senses the channel for a constant time t_{s} assumed to be much smaller than transmission times T_{on} and T_{off}. This assumption guarantees that the primary is highly unlikely to change state during the sensing period. Based on the sensing outcome, the secondary transmitter determines its own transmit power and the duration of transmission after which it has to sense the primary channel again. To formulate the optimization problem, First, we derive the capacity formula for the secondary link in the setting we mentioned before. The derivation is provided at the next chapter.

Chapter 3

Capacity of Secondary Link

In this chapter we provide formulas for the capacity for the previously stated secondary link in chapter 2. We assume that if the primary transmitter is idle, the secondary transmitter sends for time T_{F} with power P_{F}, otherwise it transmits for T_{B} with power P_{B} as shown in Figure 2.1. The power and duration chosen by the secondary transmitter depend on the channel sensing outcome and are obtained via solving a maximization problem of the weighted sum of primary and secondary throughput which will be provided at the following chapter. For the channels we assume a block fading model where the channels are known to the communicating terminals. The transmitted codewords are long enough to be affected by all channel fading states.

Following [5] and [7], we define $\delta_0(T_{\mathrm{F}})$ as the average time within a period of length T_{F} at which channel is free when it is sensed to be free at the beginning of this period. Similarly, $\delta_1(T_{\mathrm{B}})$ is the average period of time during which the channel is **free** while it is sensed to be busy and is sensed again after a time T_{B}. These two parameters are given by

$$\delta_0(t) = t - u\left(t + \frac{\exp[-(\lambda_{\mathrm{on}} + \lambda_{\mathrm{off}})t] - 1}{\lambda_{\mathrm{on}} + \lambda_{\mathrm{off}}}\right) \tag{3.1}$$

$$\delta_1(t) = (1 - u)\left(t + \frac{\exp[-(\lambda_{\mathrm{on}} + \lambda_{\mathrm{off}})t] - 1}{\lambda_{\mathrm{on}} + \lambda_{\mathrm{off}}}\right) \tag{3.2}$$

In the following two sections, we derive an upper and lower bounds on the mutual information between the input and output of the channel between the secondary terminals. It is important to note that all mutual information expressions below are conditioned on the channel gains, which are assumed to be perfectly known at the secondary receiver.

3.1 Upper Bound

We consider a genie-aided secondary receiver with knowledge of the exact pattern of primary activity. We assume that the transmitter sends two codewords, both interleaved in time. One codeword is sent successively when the channel is sensed to be free, whereas the other is sent when the channel is sensed to be busy. The analysis of the mutual information between secondary channel input and output is the same for the two codewords with appropriate use of traffic and transmission parameters. Hence, we focus here on the codeword sent successively when the channel is sensed to be free. We further assume that the time parameters, such as T_{F} and $\delta_0\left(T_{\mathrm{F}}\right)$ are all integer multiples of symbol duration, t_{s}, which is assumed to be very small relative to T_{on}, T_{off}, T_{F}, and T_{B}. Assuming the codeword is sent over m blocks each composed of $T_{\mathrm{F}}/t_{\mathrm{s}}$ samples, the total number of codeword samples, n, is equal to $mT_{\mathrm{F}}/t_{\mathrm{s}}$.

The average number of interference-free samples within a transmission block is equal to $\delta_0(T_{\mathrm{F}})/t_{\mathrm{s}}$. Applying the law of large numbers as m goes to infinity, the number of samples in the codeword that suffer from primary user interference is $n\frac{(T_{\mathrm{F}}-\delta_0(T_{\mathrm{F}}))}{T_{\mathrm{F}}}$. Let $\overline{\delta_0} = \delta_0\left(T_{\mathrm{F}}\right)/T_{\mathrm{F}}$. The number of different possible patterns for the primary activity is

$$S = \binom{n}{n\left(1-\overline{\delta_0}\right)}$$

Let $I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n)$ be the mutual information between the input sequence of length n, X_{F}^n, to the secondary channel when the primary is sensed to be inactive, and the output Y_{F}^n. Mutual information $I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n)$ is bounded by the mutual information conditioned on primary activity pattern $I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n|s)$ [10], A genie is informing the secondary user with the primary transmitter instantaneous activity. That is,

$$I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n) \leq I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n|s) \tag{3.3}$$

$$I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n|s) = \sum_l P(s=l) I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n|s=l) \tag{3.4}$$

Where the summation is over the possible interference patterns. Since the number of samples that suffer from primary interference as m goes to infinity is the same for all possible activity patterns, the term $I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n|s=l)$ is constant $\forall\ l$. Assuming Gaussian inputs,

$$I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n|s) = \log \frac{\det(I_n + \gamma_{\mathrm{F}}|g_{\mathrm{ss}}|^2 I_n + \gamma_{\mathrm{p}}|g_{\mathrm{ps}}|^2 A_l)}{\det(I_n + \gamma_{\mathrm{p}}|g_{\mathrm{ps}}|^2 A_l)} \tag{3.5}$$

Where γ_{F} and γ_{p} are the emitted powers by the secondary and primary transmitters, respectively, **normalized by** the noise variance at the secondary receiver, g_{ss} is the channel gain between the secondary transmitter and the secondary receiver, and g_{ps} is the channel gain between the primary transmitter and the secondary receiver. Matrix I_n is the $n \times n$ identity matrix, whereas A_l is an $n \times n$ diagonal matrix with ones in places corresponding to received samples during primary activity and zeros elsewhere. Recall that the number of zeros on the diagonal of A_l is the same as the number of samples that is free interference. This number equals $n\frac{\delta_0(T_{\mathrm{F}})}{T_{\mathrm{F}}} = n\overline{\delta_0}$. Combining (3.3) and (3.5), we obtain

$$I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n) \leq n\overline{\delta_0} \log(1 + \gamma_{\mathrm{F}}|g_{\mathrm{ss}}|^2) + n\left(1 - \overline{\delta_0}\right) \log(1 + \frac{\gamma_{\mathrm{F}}|g_{\mathrm{ss}}|^2}{1 + \gamma_{\mathrm{p}}|g_{\mathrm{ps}}|^2}) \tag{3.6}$$

3.2 Lower Bound

In this section we obtain a lower bound on the mutual information $I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n)$ following the analysis in [10].

$$I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n|s) - I(X_{\mathrm{F}}^n; Y_{\mathrm{F}}^n) = h(X_{\mathrm{F}}^n|s) - h(X_{\mathrm{F}}^n|Y_{\mathrm{F}}^n, s) - h(X_{\mathrm{F}}^n) + h(X_{\mathrm{F}}^n|Y_{\mathrm{F}}^n) \tag{3.7}$$

Where $h(z)$ denotes the entropy of the random variable z. Given that the input is independent of the primary interference pattern, $h\left(X_{\mathrm{F}}^{n}|s\right) = h\left(X_{\mathrm{F}}^{n}\right)$ and

$$\begin{aligned} I(X_{\mathrm{F}}^{n};Y_{\mathrm{F}}^{n}|s) - I(X_{\mathrm{F}}^{n};Y_{\mathrm{F}}^{n}) &= h(X_{\mathrm{F}}^{n}|Y_{\mathrm{F}}^{n}) - h(X_{\mathrm{F}}^{n}|Y_{\mathrm{F}}^{n}, s) \\ &= I(X_{\mathrm{F}}^{n}; s|Y_{\mathrm{F}}^{n}) = h(s|Y_{\mathrm{F}}^{n}) - h(s|X_{\mathrm{F}}^{n}, Y_{\mathrm{F}}^{n}) \end{aligned} \tag{3.8}$$

Since $h(s|X_{\mathrm{F}}^{n}, Y_{\mathrm{F}}^{n}) \geq 0$,

$$I(X_{\mathrm{F}}^{n};Y_{\mathrm{F}}^{n}|s) - I(X_{\mathrm{F}}^{n};Y_{\mathrm{F}}^{n}) \leq h(s|Y_{\mathrm{F}}^{n}) \leq h(s) \tag{3.9}$$

$$I(X_{\mathrm{F}}^{n};Y_{\mathrm{F}}^{n}) \geq I(X_{\mathrm{F}}^{n};Y_{\mathrm{F}}^{n}|s) - h(s) \tag{3.10}$$

Note that the term $I(X_{\mathrm{F}}^{n};Y_{\mathrm{F}}^{n}|s)$ is the upper bound on the mutual information which is obtained at (3.6). The term $h(s)$ represents the gap between the upper and lower bounds.

3.3 Secondary Link Capacity

When the channel is sensed to be free and codeword X_{F}^{n} is transmitted, the instantaneous capacity is given by $\lim\limits_{n\to\infty} \frac{I(X_{\mathrm{F}}^{n};Y_{\mathrm{F}}^{n})}{n}$. The upper bound on ergodic capacity, $C_{\mathrm{F}}^{\mathrm{UB}}$, is obtained from (3.6) as

$$\begin{aligned} C_{\mathrm{F}}^{\mathrm{UB}} \;&=\; \overline{\delta_0}\mathbb{E}\left[\log(1+\gamma_{\mathrm{F}}|g_{ss}|^2)\right] + \\ &\quad \left(1-\overline{\delta_0}\right)\mathbb{E}\left[\log(1+\frac{\gamma_{\mathrm{F}}|g_{ss}|^2}{1+\gamma_{\mathrm{p}}|g_{ps}|^2})\right] \end{aligned} \tag{3.11}$$

Where $\mathbb{E}\left[.\right]$ denotes the expectation operation over the channel gains.

The lower bound on capacity depends on the entropy of the interference pattern $h(s)$. The worst lower bound can be obtained by maximizing $h(s)$ assuming that all interference patterns are equally likely (i.e., uniformly distributed).

$$\begin{aligned} D_{\mathrm{F}} &= \lim_{n\to\infty} \frac{h(s)}{n} \\ &= \lim_{n\to\infty} \frac{1}{n} \log \binom{n}{n\left(1-\overline{\delta_0}\right)} \end{aligned}$$

Where D_{F} represents the gap between the upper and lower bound of capacity of the secondary link when the channel is sensed to be free. Applying Stirling's approximation to the previous equation $h(s)$ can be written as follows when n goes to ∞

$$h(s) = \log\left(\frac{e^{((n+0.5)\log(n))}}{\sqrt{(2\Pi)}(e^{(n\overline{\delta_0}+0.5)\log(n\overline{\delta_0})} + e^{(n(1-\overline{\delta_0})+0.5)\log(n(1-\overline{\delta_0}))}}\right) \tag{3.12}$$

and taking the limit, we obtain

$$D_{\mathrm{F}} = H\left(\overline{\delta_0}\right) \tag{3.13}$$

where $H(z) = -z\log z - (1-z)\log(1-z)$. The maximum value of D_{F} occurs when $\overline{\delta_0} = 0.5$ and is equal to one bit.

Doing the same steps when the channel is sensed to be busy and the second codeword X_{B}^n is sent with normalized secondary transmit power γ_{B}, the upper bound on the capacity, denoted by $C_{\mathrm{B}}^{\mathrm{UB}}$, has the exact expression as (3.11) replacing γ_{F} by γ_{B}, and $\overline{\delta_0}$ by $\overline{\delta_1} = \delta_1(T_{\mathrm{B}})/T_{\mathrm{B}}$.

$$\begin{aligned} C_{\mathrm{B}}^{\mathrm{UB}} &= \overline{\delta_1}\log(1+\gamma_{\mathrm{B}}|g_{ss}|^2) \\ &\quad + \left(1-\overline{\delta_1}\right)\log(1+\frac{\gamma_{\mathrm{B}}|g_{ss}|^2}{1+\gamma_{\mathrm{p}}|g_{ps}|^2}) \end{aligned} \tag{3.14}$$

If the output of the channel at the secondary receiver is Y_{B}^n when X_{B}^n is sent, the capacity of the channel, C_{B}, when the channel is sensed to be busy is given by

$$C_{\mathrm{B}} \geq C_{\mathrm{B}}^{\mathrm{UB}} - D_{\mathrm{B}} \tag{3.15}$$

Where D_{B}, assuming equally likely interference profiles, is equal to $H\left(\overline{\delta_1}\right)$.

Based on the channel Markov model we define P_{SS} to be the steady state probability of finding the channel free given that it is sensed again after T_{F} if sensed free, and after T_B when sensed busy. For the perfect sensing case, it can be shown in the next chapter 4.2 at page 19 that

$$P^{\mathrm{ss}} = \frac{P^{10}(T_\mathrm{B})}{1 - P^{00}(T_\mathrm{F}) + P^{10}(T_\mathrm{B})} \tag{3.16}$$

Where $P^{10}(T_\mathrm{B})$ and $P^{00}(T_\mathrm{F})$ can be obtained from (2.4) and (2.5). The channel capacity C is then

$$\begin{aligned} C &= P_{\mathrm{ss}}\frac{T_\mathrm{F}}{\mu}C_\mathrm{F} + (1 - P_{\mathrm{ss}})\frac{T_\mathrm{B}}{\mu}C_\mathrm{B} \\ &\geq P_{\mathrm{ss}}\frac{T_\mathrm{F}}{\mu}C_\mathrm{F}^{\mathrm{UB}} + (1 - P_{\mathrm{ss}})\frac{T_\mathrm{B}}{\mu}C_\mathrm{B}^{\mathrm{UB}} - D \end{aligned} \tag{3.17}$$

Parameter $\mu = P_{\mathrm{ss}}T_\mathrm{F} + (1 - P_{\mathrm{ss}})\,T_\mathrm{B}$ is the average time between sensing. The gap D between the upper and lower bounds on ergodic capacity is given by

$$D = P_{\mathrm{ss}}\frac{T_\mathrm{F}}{\mu}D_\mathrm{F} + (1 - P_{\mathrm{ss}})\frac{T_\mathrm{B}}{\mu}D_\mathrm{B} \tag{3.18}$$

Note that the limits of $\overline{\delta_0}$ and $\overline{\delta_1}$ as t goes to zero and infinity are as follows:

$$\lim_{T_\mathrm{F}\to\infty} \overline{\delta_0} = 1 - u$$

$$\lim_{T_\mathrm{F}\to 0} \overline{\delta_0} = 1$$

$$\lim_{T_\mathrm{B}\to\infty} \overline{\delta_1} = 1 - u$$

$$\lim_{T_\mathrm{B}\to 0} \overline{\delta_1} = 0$$

This leads us to obtain the worst capacity gap. This gap can be obtained by taking the limit for D at (3.18).

$$\lim_{T_\mathrm{F},T_\mathrm{B}\to\infty} D = H(1-u) = H(u) \tag{3.19}$$

3.4 numerical results

In this section we provide some numerical results for the gap between the upper and lower bounds on the capacity. We use $T_{\text{off}} = 5$ (certain units of time) and sweep the value of u from 0 to 1. We present here two cases corresponding to small and large T_{F} and T_{B}, namely, $T_{\text{F}} = 1$ and $T_{\text{B}} = 2$ for first case, and $T_{\text{F}} = 20$ and $T_{\text{B}} = 15$ for the second. It is shown in Figure 3.1 that the gap is lower than $H(u)$ for small values of transmission times. On the other hand, when T_{F} and T_{B} are large in value, the gap is exactly equal to $H(u)$. It can be easily shown that both $\overline{\delta_0}$ and $\overline{\delta_1}$ converge to $1 - u$ as T_{F} and T_{B} go to infinity. The maximum of the gap in this case is 1 bit/channel per channel use when $u = 0.5$.

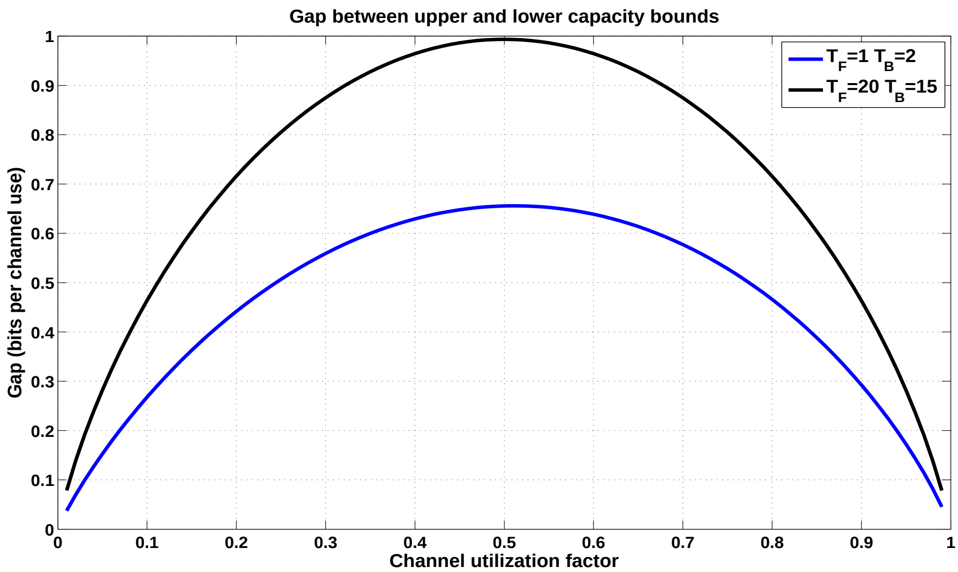

Figure 3.1: Gap D between lower and upper bounds as a function of u. For the case $T_{\text{F}} = 20$ and $T_{\text{B}} = 15$, the gap is almost $H(u)$. For $T_{\text{F}} = 1$ and $T_{\text{B}} = 2$, the gap is smaller.

Chapter 4

Joint Optimization Problem

In this chapter, we explain the problem of finding the optimal secondary transmission time and power given the outcome of the sensing process.

4.1 Problem Formulation

We formulate the cognitive power and transmission time control problem as an optimization problem with the objective of maximizing a weighted sum of the primary, R_{p}, and secondary, R_{s}, rates. Specifically, we seek to maximize $\mathbb{E}\{(1-\alpha)\, R_{\mathrm{s}} + \alpha R_{\mathrm{p}}\}$, where $\mathbb{E}\{.\}$ denotes the expectation operation over the sensing outcome and primary activity. The constant $\alpha \in [0, 1]$ is chosen on the basis of the required primary throughput. In order to protect the primary user from interference and service interruption, parameter α should be close to one. In the sequel, however, we study the full range of α so that our results account for other cases where there is no clear priority among the users. The constraints of the optimization problem are that the secondary power lies in the interval $[0, P_{\mathrm{max}}]$, and that the time between sensing operations exceeds t_{s}. The problem is generally non-convex and, consequently, we resort to exhaustive search to obtain the solution when the number of optimization parameters is small.

In this thesis, we consider two sensing scenarios: 1) perfect sensing, and 2) soft sensing where the cognitive transmitter uses some sensing metric γ, say the output of an energy detector, to determine its transmission parameters. Under the soft sensing mode of operation, the range of values of γ is divided into intervals and the transmission power and time are determined based on the interval on which the actual sensing metric γ lies. The parameters to optimize the rate objective function are the transmission powers and times corresponding to each interval and also the boundaries between intervals.

We assume that the primary link is in outage whenever the primary rate $r_\circ$ exceeds the capacity of the primary channel. The primary outage probability when the secondary transmitter emits power p is given by:

$$P_\circ(p) = \Pr\left\{ r_\circ > \log\left(1 + \frac{P_\mathrm{p}\, g_\mathrm{pp}}{p\, g_\mathrm{sp} + \sigma_\mathrm{p}^2}\right)\right\} \tag{4.1}$$

Where σ_p^2 is the noise variance of the primary receiver. The expression of $P_\circ(p)$ for Rayleigh fading channels is given in the Appendix. We assume that the channel gains vary slowly over time and are almost constant over several epochs of primary and secondary transmission. We will provide expression for $P_\circ(p)$ in the Appendix.

For the secondary rate, we assume that the secondary receiver tracks the instantaneous capacity of the channel and, hence, the maximum achievable rate is obtained by averaging over the channel gains and interference levels [[13], equation 8]. The ergodic capacity of the secondary channel when the cognitive transmitter emits power p and the primary transmitter is off is expressed as

$$C_\circ(p) = \mathbb{E}_{g_\mathrm{ss}}\left\{\log\left(1 + \frac{p\, g_\mathrm{ss}}{\sigma_\mathrm{s}^2}\right)\right\} \tag{4.2}$$

where σ_s^2 is the noise variance of the secondary receiver. When there is simultaneous primary and secondary transmissions, the ergodic capacity of the secondary channel becomes

$$C_1(p) = \mathbb{E}_{g_\mathrm{ss}, g_\mathrm{ps}}\left\{\log\left(1 + \frac{p\, g_\mathrm{ss}}{P_\mathrm{p} g_\mathrm{ps} + \sigma_\mathrm{s}^2}\right)\right\} \tag{4.3}$$

We also provide expressions for $C_\circ(p)$ and $C_1(p)$ in the Appendix.

4.2 Perfect Sensing

We mean by perfect sensing that the state of the channel, whether vacant or occupied, is known without error after the channel is sensed. The four parameters used to maximize the weighted sum throughput are P_F and T_F defined as the power and transmission time when the primary channel is free, and P_B and T_B corresponding to the busy primary state. Before formulating the optimization problem under perfect sensing, we need to introduce several parameters that pertain to the primary traffic. The probability, π_m, that the mth observation of the channel occurs when the channel is free can be calculated using Markovian property of the traffic model.

$$\pi_m = \pi_{m-1} P^{00}\left(t_\mathrm{s} + T_\mathrm{F}\right) + \left(1 - \pi_{m-1}\right) P^{10}\left(t_\mathrm{s} + T_\mathrm{B}\right) \tag{4.4}$$

Another parameter is P^{ss} which is the steady state fraction of time the channel is free when sensed according to some scheme. In the perfect sensing scheme, the channel, when sensed free, is sensed again after $t_\mathrm{s} + T_\mathrm{F}$. When sensed busy, it is sensed again after $t_\mathrm{s} + T_\mathrm{B}$. Parameter P^{ss} can be obtained by setting $\pi_m = \pi_{m-1} = P^{\mathrm{ss}}$ in (4.4) to get

$$P^{\mathrm{ss}} = \frac{P^{10}(t_\mathrm{s} + T_\mathrm{B})}{1 - P^{00}(t_\mathrm{s} + T_\mathrm{F}) + P^{10}(t_\mathrm{s} + T_\mathrm{B})} \tag{4.5}$$

The average time between sensing times is given by

$$\mu = P^{\mathrm{ss}}\left(t_\mathrm{s} + T_\mathrm{F}\right) + \left(1 - P^{\mathrm{ss}}\right)\left(t_\mathrm{s} + T_\mathrm{B}\right) \tag{4.6}$$

Finally, we also need the average time the channel is free during a period of t units of time if sensed to be free. We denote this quantity by $\delta^\circ(t)$ and is given by (3.1). On the other hand, if the channel is sensed to be busy, the average time the channel is free during a period of t units of time is given by (3.2).

The secondary throughput averaged over primary activity is given by

$$\begin{aligned}\overline{R}_{\mathrm{s}} &= P^{\mathrm{ss}}\frac{\delta^{\circ}\left(T_{\mathrm{F}}\right)}{\mu}C_{\circ}\left(P_{\mathrm{F}}\right)+ \\ &\quad P^{\mathrm{ss}}\frac{T_{\mathrm{F}}-\delta^{\circ}\left(T_{\mathrm{F}}\right)}{\mu}C_{1}\left(P_{\mathrm{F}}\right)+ \\ &\quad \left(1-P^{\mathrm{ss}}\right)\frac{\delta^{1}\left(T_{\mathrm{B}}\right)}{\mu}C_{\circ}\left(P_{\mathrm{B}}\right)+ \\ &\quad \left(1-P^{\mathrm{ss}}\right)\frac{T_{\mathrm{B}}-\delta^{1}\left(T_{\mathrm{B}}\right)}{\mu}C_{1}\left(P_{\mathrm{B}}\right)\end{aligned} \tag{4.7}$$

The first two terms in the above expression are the secondary throughput obtained if the primary is inactive when the channel is sensed. When the sensing outcome is that the channel is free, the secondary emits power P_{F} for a duration T_{F}. During the secondary transmission period, the primary transmitter may resume activity. The average amount of time the primary remains idle during a period of length T_{F} after the channel is sensed to be free is obtained by using $t = T_{\mathrm{F}}$ in (3.1). This is the duration of secondary transmission free from interference from the primary transmitter. On the other hand, the primary transmits during secondary operation for an average period of $T_{\mathrm{F}} - \delta^{\circ}\left(T_{\mathrm{F}}\right)$. The last two terms in (4.7) are the same as the first two but when the channel is sensed to be busy. In this case, the transmit secondary power is P_{B} and the transmission time is T_{B}, of which a duration of $\delta^{1}\left(T_{\mathrm{B}}\right)$ is free, on average, from primary interference.

The primary throughput is given by

$$\begin{aligned}\overline{R}_{\mathrm{p}} &= r_{\circ}P^{\mathrm{ss}}\frac{T_{\mathrm{F}}-\delta^{\circ}\left(T_{\mathrm{F}}\right)}{\mu}\left[1-P_{\circ}\left(P_{\mathrm{F}}\right)\right]+ \\ &\quad r_{\circ}\left(1-P^{\mathrm{ss}}\right)\frac{T_{\mathrm{B}}-\delta^{1}\left(T_{\mathrm{B}}\right)}{\mu}\left[1-P_{\circ}\left(P_{\mathrm{B}}\right)\right]\end{aligned} \tag{4.8}$$

We ignore the primary throughput that may be achieved during the sensing period because t_{s} is assumed to be much smaller than $T_{\mathrm{on}}$and T_{off}. The two terms of (4.8) correspond to the sensing outcomes of the channel being free and busy, respectively.

The optimization problem can then be written as

Find: $T_{\mathrm{F}}, T_{\mathrm{B}}, P_{\mathrm{F}}$ and P_{B} That maximize:

$(1-\alpha)\ \overline{R}_{\mathrm{s}}(T_{\mathrm{F}}, T_{\mathrm{B}}, P_{\mathrm{F}}, P_{\mathrm{B}}) + \alpha\ \overline{R}_{\mathrm{p}}(T_{\mathrm{F}}, T_{\mathrm{B}}, P_{\mathrm{F}}, P_{\mathrm{B}})$

Subject to:

$T_{\mathrm{F}} \geq 0,\ T_{\mathrm{B}} \geq 0,\ 0 \leq P_{\mathrm{F}} \leq P_{\max}$ and $0 \leq P_{\mathrm{B}} \leq P_{\max}$

4.3 Soft Sensing

Soft sensing means that the sensing metric is used directly to determine the secondary transmission power and duration. In the sequel, we re-formulate the weighted sum throughput optimization problem assuming quantized soft sensing, where the sensing metric, from a matched filter or an energy detector for instance, is quantized before determining the power and duration of transmission. Let γ be the sensing metric with the known conditional pdfs: $f_\circ(\gamma)$ given that the primary is in the idle state and $f_1(\gamma)$ conditioned on the primary transmitter being active. We assume that the number of quantization levels is $S+1$. The kth level extends from threshold γ_{k-1}^{th} to γ_k^{th} assuming that $\gamma_0^{\text{th}} = 0$ and $\gamma_{S+1}^{\text{th}} = \infty$. The probability that the metric γ is between γ_{k-1}^{th} and γ_k^{th} when the primary channel is free is given by

$$\begin{aligned} \epsilon_k &= \Pr\{\gamma_{k-1}^{\text{th}} \leq \gamma \leq \gamma_k^{\text{th}} | \text{channel is free}\} \\ &= \int_{\gamma_{k-1}^{\text{th}}}^{\gamma_k^{\text{th}}} f_0(\gamma)\, d\gamma \end{aligned} \tag{4.9}$$

where $k = 1, 2, \cdots (S+1)$. On the other hand, The probability that γ is between γ_{k-1}^{th} and γ_k^{th} when the primary channel is busy is given by

$$\begin{aligned} \vartheta_k &= \Pr\{\gamma_{k-1}^{\text{th}} \leq \gamma \leq \gamma_k^{\text{th}} | \text{channel is busy}\} \\ &= \int_{\gamma_{k-1}^{\text{th}}}^{\gamma_k^{\text{th}}} f_1(\gamma)\, d\gamma \end{aligned} \tag{4.10}$$

When γ is between γ_{k-1}^{th} and γ_k^{th}, the secondary transmitted power is P_k and the duration of transmission is T_k.

As in the perfect sensing case, the probability that mth observation of the channel happens when the channel is free, denoted by π_{m}, can be calculated using Markovian property of the channel model.

$$\begin{aligned} \pi_m &= \pi_{m-1}\sum_{k=1}^{S+1}\epsilon_k P^{00}(t_\mathrm{s}+T_k) \\ &\quad +(1-\pi_{m-1})\sum_{k=1}^{S+1}\vartheta_k P^{10}(t_\mathrm{s}+T_k) \end{aligned} \tag{4.11}$$

At steady state, $\pi_{m-1}=\pi_m$ and the steady state probability of sensing the channel while it is free becomes

$$P^{\mathrm{ss}} = \frac{\sum_{k=1}^{S+1}\vartheta_k P^{10}(t_\mathrm{s}+T_k)}{1-\sum_{k=1}^{S+1}\epsilon_k P^{00}(t_\mathrm{s}+T_k)+\sum_{k=1}^{S+1}\vartheta_k P^{10}(t_\mathrm{s}+T_k)} \tag{4.12}$$

The average time between sensing events is given by

$$\mu = P^{\mathrm{ss}}\sum_{k=1}^{S+1}\epsilon_k\left(t_\mathrm{s}+T_k\right)+\left(1-P^{\mathrm{ss}}\right)\sum_{k=1}^{S+1}\vartheta_k\left(t_\mathrm{s}+T_k\right) \tag{4.13}$$

The mean secondary throughput averaged over the primary activity and the sensing metric is given by

$$\begin{aligned} \overline{R}_\mathrm{s} &= P^{\mathrm{ss}}\sum_{k=1}^{S+1}\epsilon_k\left[\frac{\delta^\circ(T_k)}{\mu}C_\circ\left(P_k\right)+\frac{T_k-\delta^\circ(T_k)}{\mu}C_1\left(P_k\right)\right] \\ &\quad +(1-P^{\mathrm{ss}})\sum_{k=1}^{S+1}\vartheta_k\Big[\frac{\delta^1(T_k)}{\mu}C_\circ\left(P_k\right)+\frac{T_k-\delta^1(T_k)}{\mu}C_1\left(P_k\right)\Big] \end{aligned} \tag{4.14}$$

The mean primary throughput is

$$\begin{aligned} \overline{R}_\mathrm{p} &= r_\circ P^{\mathrm{ss}}\sum_{k=1}^{S+1}\epsilon_k\frac{T_k-\delta^\circ(T_k)}{\mu}\left[1-P_\circ\left(P_k\right)\right]+ \\ &\quad r_o(1-P^{ss})\sum_{k=1}^{S+1}\vartheta_k\frac{T_k-\delta^1(T_k)}{\mu}\left[1-P_\circ\left(P_k\right)\right] \end{aligned} \tag{4.15}$$

4.4 Traffic Parameters Learning and Estimating

The traffic parameters of the primary network can be learned by probing the channel for a specified learning period without transmission. The sensing outcome can be used to estimate the unknown parameters. This part was investigated by [23]. In the case of perfect sensing, a maximum likelihood estimator can be employed [5]. The parameters λ_{on} and λ_{off} are obtained via maximizing the likelihood function

$$f\left(S_1, S_2, S_3, \ldots S_{\text{L}} | \, \lambda_{\text{on}}, \lambda_{\text{off}}\right) \tag{4.16}$$

where L is the number of sensing outcomes obtained during the learning phase, and S_i is the ith sensing outcome which has one of two values: $S_i = 0$ if the channel is sensed to be free, and $S_i = 1$ for a busy sensing outcome. Using the Markovian property, the likelihood function (4.16) can be written as

$$f(S_1)f(S_2|S_1)f(S_3|S_2)\ldots f(S_{\text{L}}|S_{\text{L}-1}) \tag{4.17}$$

Where $f(S_i = v|S_{i-1} = w)$ is the transition probability $P^{wv}\left(\tau_L\right)$ defined above with $v \in \{0, 1\}$, $w \in \{0, 1\}$, and τ_L is the time between two sensing events. In the simulation section, we present a curve showing the impact of using the learned rather than the true primary traffic parameters. It is important to mention that parameter learning is not the main focus of this work.

4.5 Numerical Results

In this section we present simulation results for the perfect and soft sensing schemes discussed in this chapter. The weighted sum rate maximization problem is non-convex, hence, we do exhaustive search to obtain the optimal parameters. The parameters used in our simulations presented here are: $T_{\text{on}} = 4$, $T_{\text{off}} = 5$, $t_{\text{s}} = 0.05$, $r_\circ = 4.5$ nats, $\sigma_{\text{s}}^2 = \sigma_{\text{p}}^2 = 1$, $P_{\text{p}} = 100$, $P_{\max} = 10$, $\overline{g}_{\text{ss}} = 2$, $\overline{g}_{\text{pp}} = 3$, and $\overline{g}_{\text{ps}} = .03$. In order to do the exhaustive search, we have imposed an artificial upper bound on transmission time equal to 20. We analyze the results for perfect sensing in Subsection 4.5.1 and for soft sensing in Subsection 4.5.2. The parameters for channels **A** and **B** used in the analysis are the same except for $\overline{g}_{\mathrm{sp}}$ which is equal to 2 for channel **A** and 0.2 for channel **B**.

4.5.1 Perfect Sensing

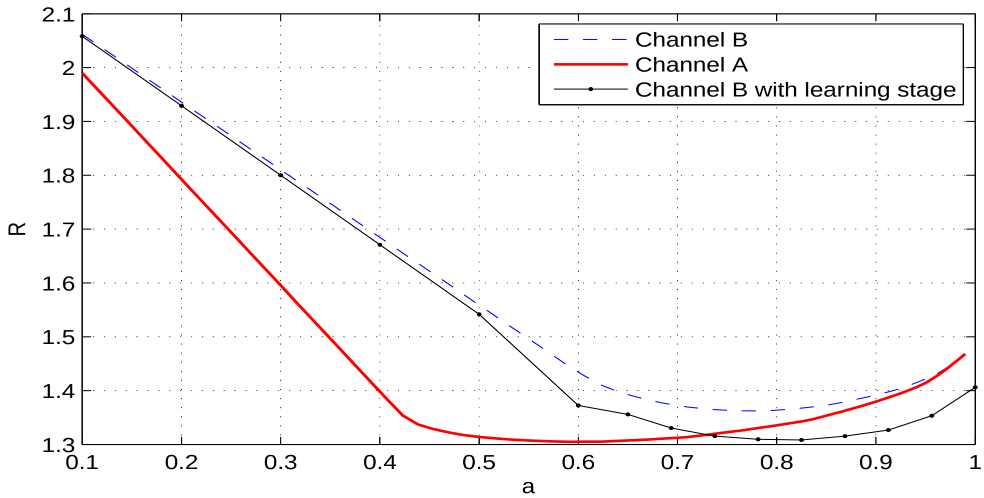

Figure 4.1: Perfect sensing weighted sum throughput versus α for channels **A** and **B**.

The weighted sum throughput versus α is shown in Figure 4.1 for channels **A** and **B**, whereas the rate region depicting the variation of secondary with primary throughput is provided in Figure 4.2. It is clear from Figure 4.1 that as the gain $\overline{g}_{\mathrm{sp}}$ increases, the level of interference at the primary receiver increases leading to lower data rates. We also include here the curve for the mean weighted sum throughput for channel **B** when the traffic parameters λ_{on} and λ_{off} are estimated. The learning parameters (explained in Section 2) are $L = 25$ and $\tau_{\mathrm{L}} = 0.5$. It is clear from the figure that there is a degradation in weighted sum throughput due to the uncertainty regarding the traffic parameters. As we have emphasized earlier, learning is not the main focus of this paper, but will be the subject of future investigation.

The optimal transmission power and time parameters for channel **A** are given in

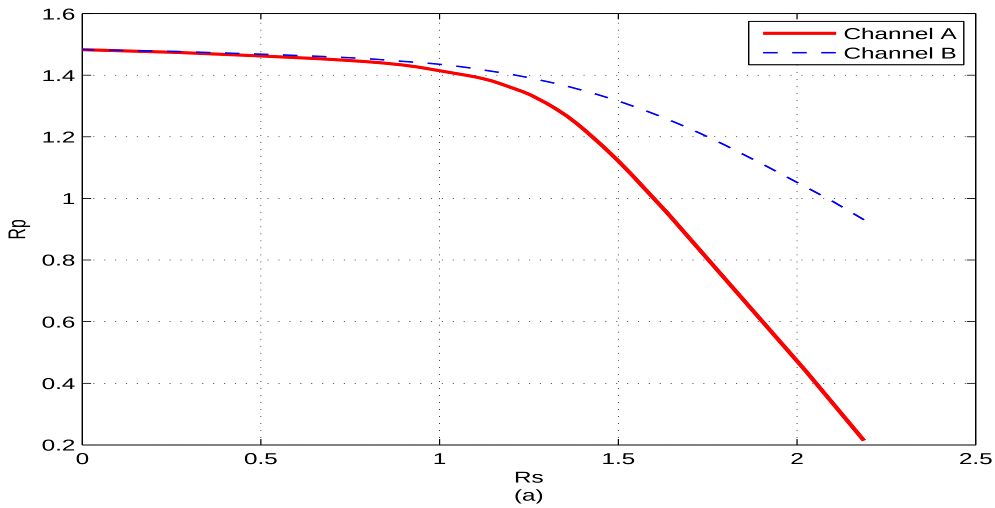

Figure 4.2: Rate region, $\mathrm{R_s}$ vs. $\mathrm{R_p}$ for channels **A** and **B**.

Figure 4.3. For small α value, which corresponds to giving more importance to the secondary throughput, the secondary transmitter emits $P_{\max}$ whether the channel is sensed to be free or busy. The transmission time for both sensing outcomes are the maximum possible. Recall that this maximum is artificial and is imposed by the exhaustive search solution. In fact, for α approaching zero, the secondary transmitter sends with $P_{\max}$ continuously without the need to sense the channel again. If the optimal $P_{\mathrm{F}} = P_{\mathrm{B}}$, then sensing becomes superfluous because the exact same power would be used regardless of the sensing outcome. As α increases, the power transmitted when the channel is sensed to be busy is reduced below $P_{\max}$. In addition, the transmission times are reduced for more frequent checking of primary activity. As α approaches unity, the secondary transmitter is turned off and the channel is not sensed. Figure 4.4 gives the optimal transmission parameters for channel **B**. It is evident from the figure that as the level of interference from secondary transmitter to primary receiver is decreased, P_{B} becomes lower than $P_{\max}$ at a higher α compared to **A**. If we make $\overline{g}_{\mathrm{sp}} = 0.002$, the secondary transmits all time with maximum power regardless of the sensing outcome. This is shown in Figure 4.5.

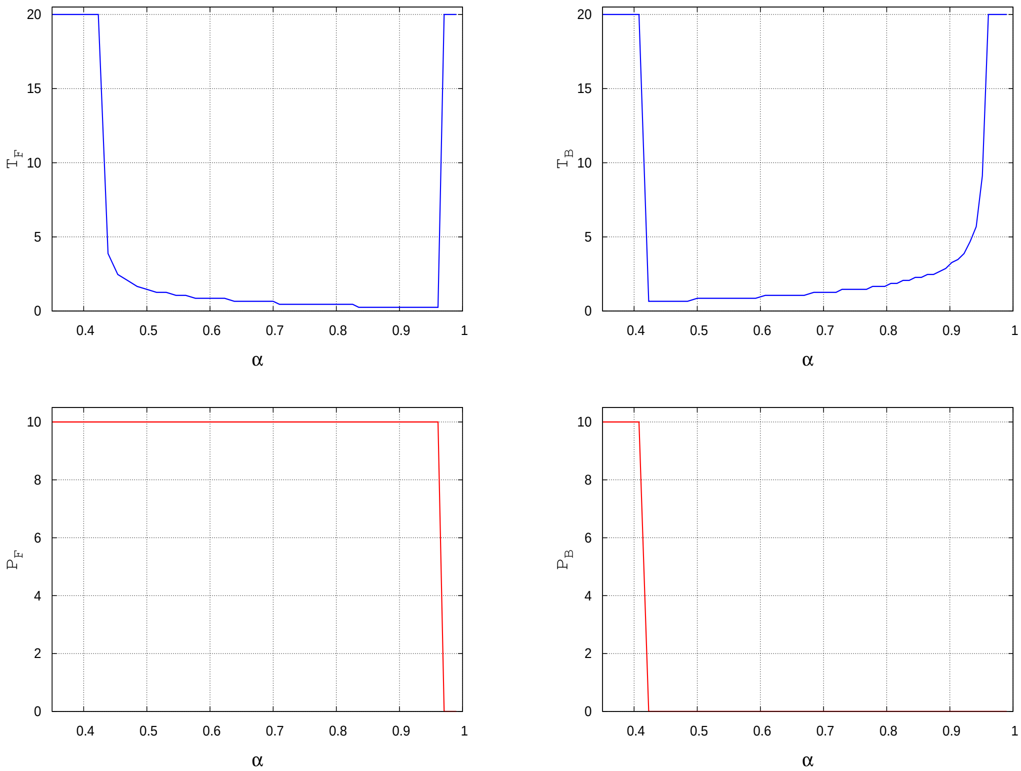

Figure 4.3: Perfect sensing power and transmission time results for channel **A**.

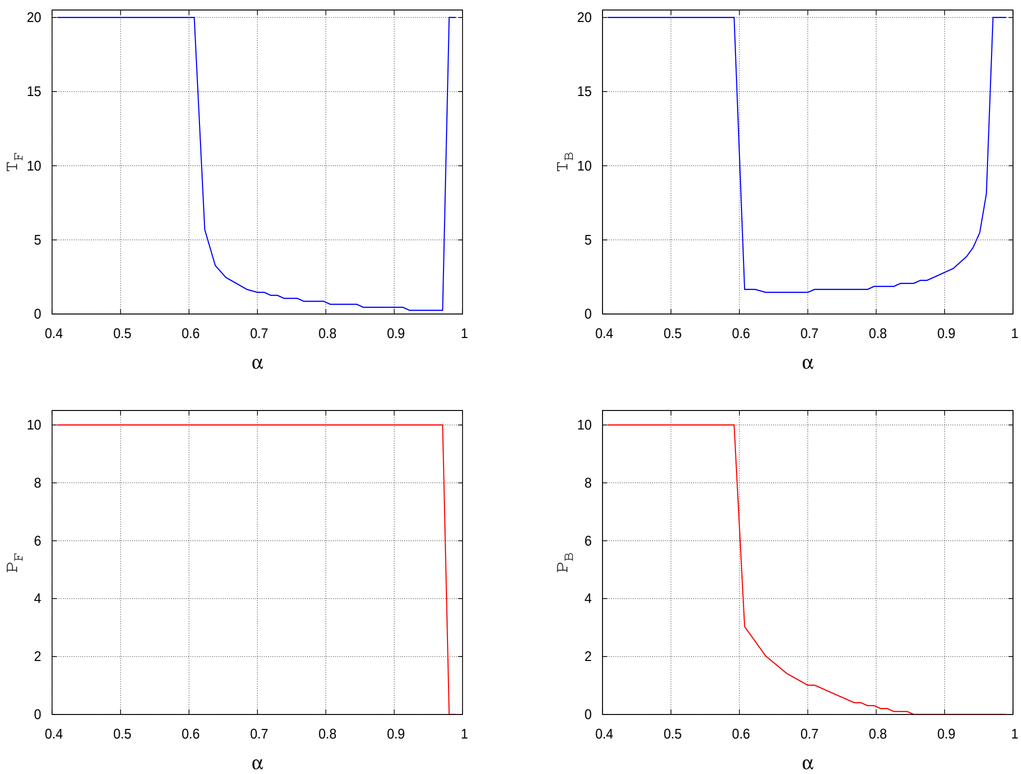

Figure 4.4: Perfect sensing power and transmission time results for channel **B**.

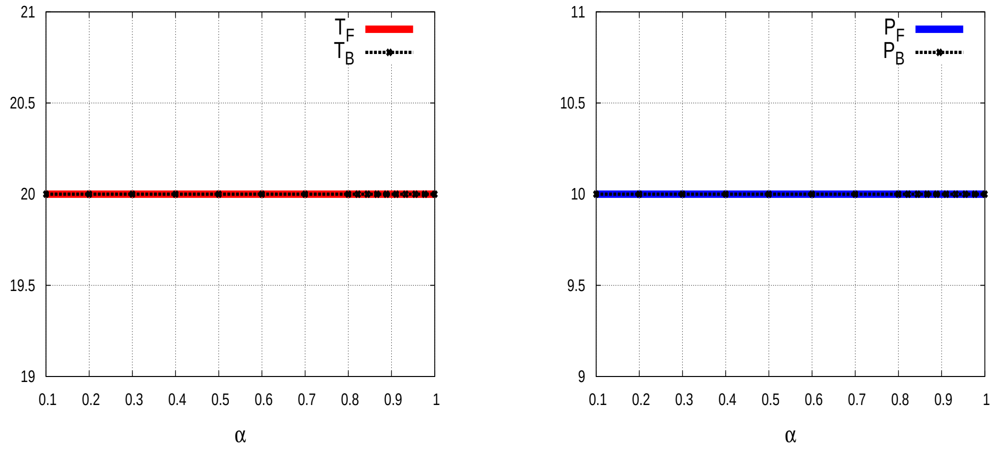

Figure 4.5: Perfect sensing power and transmission time results for g_{sp}=0.002.

4.5.2 Soft Sensing

In the soft sensing case, the optimization parameters are $2(S+1)$ transmission powers and times corresponding to each quantization level. There are also S thresholds defining the boundaries of the quantization levels. Hence, the total number of parameters is $3S+2$. The conditional distributions of the sensing metric γ used in the simulations are $f_\circ(\gamma) = \exp(-\gamma)$ and $f_1(\gamma) = \exp(-[\gamma+\gamma_\circ])\ I_\circ\left(2\sqrt{\gamma\gamma_\circ}\right)$, where $I_\circ$ is the zero order modified Bessel function and $\gamma_\circ$ is a parameter related to the mean value of $f_1(\gamma)$. We present here the results for one and two thresholds. The case of one threshold corresponds to the imperfect sensing case where the primary is assumed to be active when γ exceeds some threshold and inactive otherwise. The false alarm probability is given by ϵ_2, whereas the miss detection probability is ϑ_1. Figure 4.6 give the optimal parameters as a function of α and for $\gamma_\circ = 3$. As is evident from the figure, the optimal threshold decreases with α. Under the imperfect sensing interpretation of the one threshold case, this means that as α increases putting more emphasis on the primary rate, the required false alarm probability is increased while the miss detection probability is decreased to reduce the chance of collision with the primary user. Figure 4.7 shows the weighted sum throughput using one and two thresholds for channel $\mathbf{B}$ and $\gamma_\circ = 3$. There is a range of α values for which the two-threshold scheme improves very slightly the weighted sum rates.

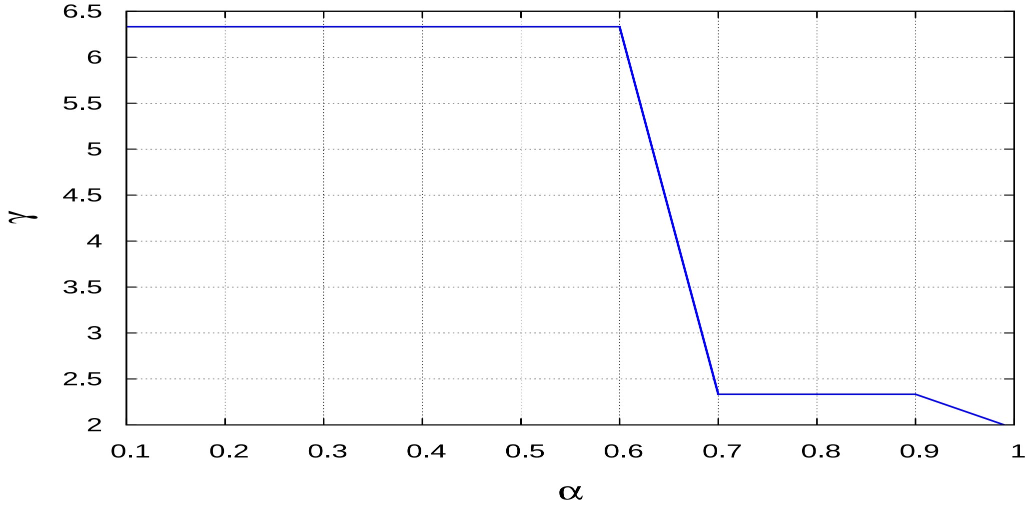

Figure 4.6: Soft sensing optimal threshold for channel **B**.

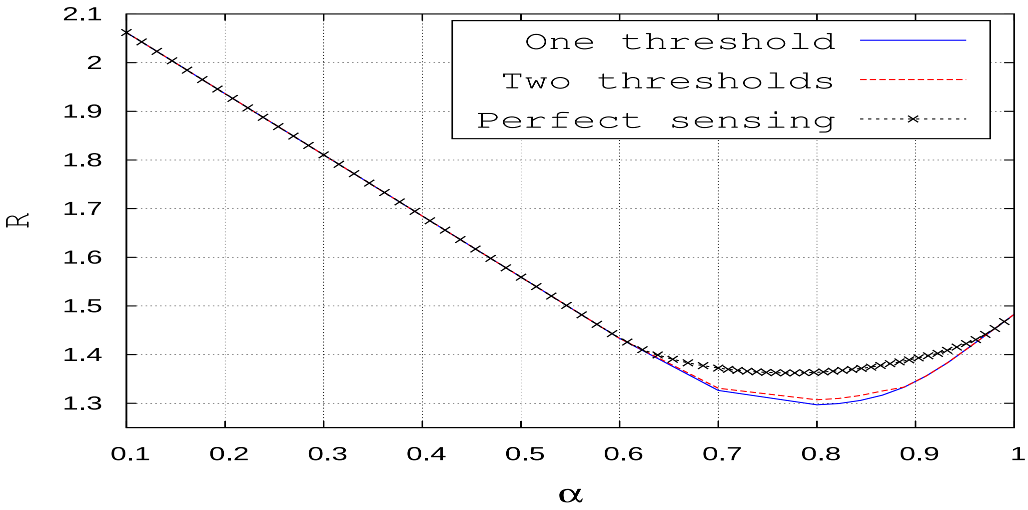

Figure 4.7: Soft sensing weighted sum throughput versus α using one and two thresholds for channel **B**. The results from perfect sensing is provided for comparison.

Chapter 5

Power Control using Feedback of Primary Link

The previous scenario that we have discussed in the previous chapters depends on channel sensing and, hence, is often called listen before talk (LBT). Most existing works apply LBT concept because of its simplicity. This concept, however, focuses on the primary transmitter rather than the receiver whose protection from interference is the main concern. The problems of LBT are as follows [19]:

- It presumes the worst case fading environment.
- It does not allow SU systems to explore the extra capacity when a PU system is not fully loaded and can tolerate more interference.
- Conventional spectrum sensing suffers from the well known hidden terminal problem.

In order to avoid these disadvantages, we adopt another scheme based on the primary feedback (ACKs and NACKs) similar to [19], [20], [21] and [22]. The proposed scheme exploits the feedback from the primary receiver to the primary transmitter. This kind of feedback is available in typical two-way primary networks such as WiMAX, CDMA cellular systems and WiFi networks. Secondary user can exploit the information embedded in the feedback message. Using this information enables the SU to ascertain its impact on the primary receiver.

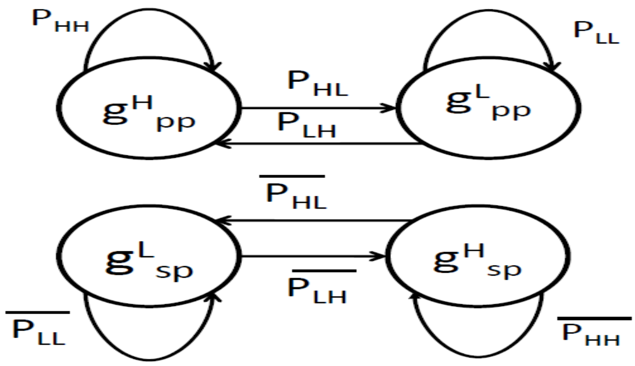

Figure 5.1: Markovian model for the channel

5.1 Proposed system model

Our model is very similar to the model in [20] and [21]. We are considering a two-way primary communication system. The primary transmitter sends its data over a forward channel. The primary receiver responds with an ACK or NACK to indicate if the transmission has been successful.

Primary transmission is assumed to be slotted and the primary user sends with a fixed power P_{P} and a fixed rate r_P. The channel between primary transmitter and primary receiver, g_{pp}, is assumed to follow a two-state Markov chain model as shown in Figure 5.1. The two states correspond to two g_{pp} values denoted g_{pp}^H and g_{pp}^L, where $g_{pp}^H > g_{pp}^L$. The two values can be chosen arbitrarily. For instance, one of them may be relatively high corresponding to a high link quality, and the other relatively low indicating a poor link quality. A high link quality implies that, without significant interference, the transmission is likely to be decoded correctly and, consequently, an ACK is emitted by the receiver. Let q_1 be the probability that $g_{pp} = g_{pp}^H$, and q_2 be the probability that $g_{sp} = g_{sp}^L$. P_{HH}, P_{HL}, P_{LH} and P_{LL} indicate the transition probabilities of the g_{pp} Markov chain, whereas $\overline{P_{HH}}$, $\overline{P_{HL}}$, $\overline{P_{LH}}$ and $\overline{P_{LL}}$ denote the transition probabilities of the g_{sp} model. The two channels and their transitions are independent. Our work is different from the work in [20]. The secondary user is listening for these ACK/NACK packets and exploits this information to control the power not only to decide the admission policy. In [19] and [21] a power control policy is investigated but the system model is not Markovian.

5.2 Problem Formulation

The objective of the secondary user in our work is to choose the optimum power to maximize the value of the accumulated reward. The immediate reward is the weighted sum primary and secondary throughput. It is given by:

$$r(k) = (1-\alpha)(\log(1+\frac{P_s(k)\left|g_{ss}\right|^2}{\sigma_s^2+\left|g_{ps}\right|^2 P_{\mathrm{P}}})) + \alpha \times r_P \times P_{ACK} \tag{5.1}$$

where $P_s(k)$ is the power transmitted by secondary user at time slot k, α is the weight that defines how much protection is afforded to the primary user, and P_{ACK} is the probability of correct reception by the primary receiver. P_{ACK} is given by:

$$P_{ACK} = q1q2\gamma_1 + q1(1-q2)\gamma_2 + (1-q1)q2\gamma_3 + (1-q1)(1-q2)\gamma_4 \tag{5.2}$$

factors γ_1, γ_2, γ_3 and γ_4 are respectively defined as $P(ACK \mid g_{pp} = g_{pp}^H \,\&\, g_{sp} = g_{sp}^L)$, $P(ACK \mid g_{pp} = g_{pp}^L \,\&\, g_{sp} = g_{sp}^L)$, $P(ACK \mid g_{pp} = g_{pp}^H \,\&\, g_{sp} = g_{sp}^H)$ and $P(ACK \mid g_{pp} = g_{pp}^L \,\&\, g_{sp} = g_{sp}^H)$. The objective function can then be written as follows:

$$V(q1,q2) = \max_{\pi}\left[\mathbb{E}\Big(\sum_{k=0}^{\infty} w^k r(q_1(k), q_2(k), P_s(k))\Big) \mid q_1(0) = q_1, q_2(0) = q_2\right] \tag{5.3}$$

where $\mathbb{E}$ denotes the expectation on the random ACKs and NACKs received and π is the optimum policy, i.e., the optimum secondary power chosen at each time slot. Finally, parameter w is discount factor, $0 \leq w < 1$.

Each time slot the channel state probability of next time slot is updated based on the received feedback at the current time slot as follows.

$$\Psi_{ACK}(K) = P(g_{pp}(k) = g_{pp}^H|ACK) = \frac{q_1(k)(\gamma_1 q_2(k) + \gamma_2(1-q_2(k)))}{P_{ACK}} \tag{5.4}$$

$$\Psi_{NACK}(K) = P(g_{pp}(k) = g_{pp}^{H}|NACK) = \frac{q_1(k)((1-\gamma_1)q_2(k) + (1-\gamma_2)(1-q_2(k)))}{1-P_{ACK}} \tag{5.5}$$

using (5.4) and (5.5) we can get the updated state probability as follows:

$$\begin{aligned} q_1(k+1) &= P(g_{pp}(k+1) = g_{pp}^{H}|\text{ACK in current time slot}) \\ &= P_{HH}\Psi_{ACK} + P_{LH}(1-\Psi_{ACK}) \end{aligned} \tag{5.6}$$

$$\begin{aligned} \overline{q_1}(k+1) &= P(g_{pp}(k+1) = g_{pp}^{H}|\text{NACK in current time slot}) \\ &= P_{HH}\Psi_{NACK} + P_{LH}(1-\Psi_{NACK}) \end{aligned} \tag{5.7}$$

Same argument can be used for updating the g_{sp} state probability.

$$\overline{\Psi}_{ACK}(K) = P(g_{sp}(k) = g_{sp}^{L}|ACK) = \frac{q_2(k)(\gamma_1 q_1(k) + \gamma_3(1-q_1(k)))}{P_{ACK}} \tag{5.8}$$

$$\overline{\Psi}_{NACK}(K) = P(g_{sp}(k) = g_{sp}^{L}|NACK) = \frac{q_2(k)((1-\gamma_1)q_1(k) + (1-\gamma_3)(1-q_1(k)))}{1-P_{ACK}} \tag{5.9}$$

$$\begin{aligned} q_2(k+1) &= P(g_{sp}(k+1) = g_{sp}^{L}|\text{ACK in current time slot}) \\ &= \overline{\Psi}_{ACK}\overline{P_{LL}} + \overline{P_{HL}}(1-\overline{\Psi}_{ACK}) \end{aligned} \tag{5.10}$$

$$\begin{aligned} \overline{q_2}(k+1) &= P(g_{sp}(k+1) = g_{sp}^{L}|\text{NACK in current time slot}) \\ &= \overline{\Psi}_{NACK}\overline{P_{LL}} + \overline{P_{HL}}(1-\overline{\Psi}_{NACK}) \end{aligned} \tag{5.11}$$

5.3 Choosing Optimum Policy

Expression (5.3) satisfies the following Bellman equation [26]

$$\begin{aligned} V(q1(k), q2(k)) &= \max_{P_s(k)\in[0,P_{max}]}[r + wP_{ACK}V(q_1(k+1), q_2(k+1)) \\ &+ w(1-P_{ACK})V(\overline{q_1}(k+1), \overline{q_2}(k+1))] \end{aligned} \tag{5.12}$$

where P_{max} is the maximum power allowed to be sent by the secondary user. Simulation results are provided for the following parameters using MATLAB. We assume capacity achieving code. As a result, for each state we have γ_1, γ_2, γ_3 and γ_4 either equal one or zero. If the corresponding γ equal 0, this means that primary is on outage. Recall that probability of correct receiving at the primary receiver depends on the power sent by the secondary user. Simulations are done where the protection weight $\alpha = 0.5$ and discount factor $w = 0.5$. Maximum power allowed by secondary user, P_{max}, equal 20.

Simulations are done for two channels. Each channel has different values for the states of the twi channels. Channel A has $g_{pp}^H = 5$, $g_{pp}^L = 3$, $g_{sp}^L = 0.2$ and $g_{sp2}^H = 1$. Channel B has $g_{pp}^H = 3$, $g_{pp}^L = 0.5$, $g_{sp}^L = 0.5$ and $g_{sp2}^H = 13$. Provided figures show the optimum power and value of the objective function versus different values of Q_1 and Q_2.

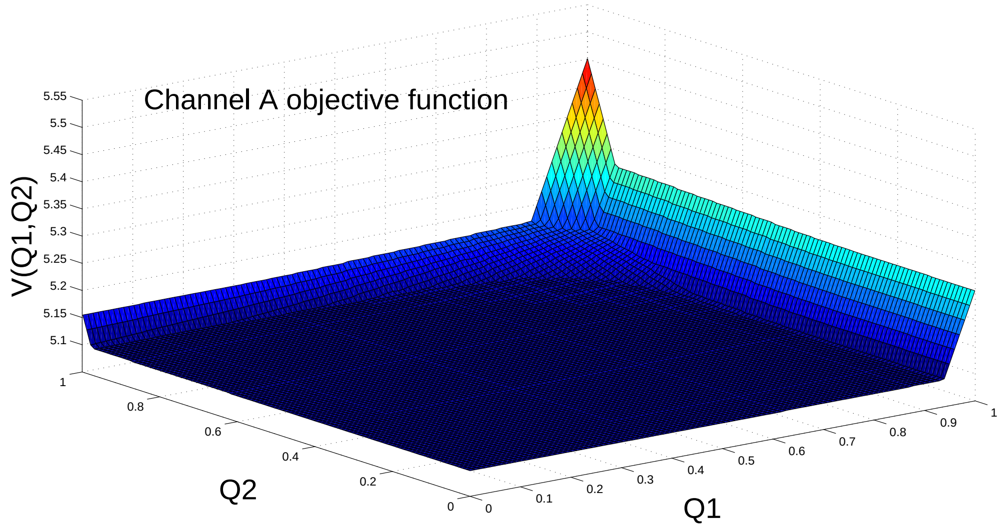

Figure 5.2: Objective function for channel A

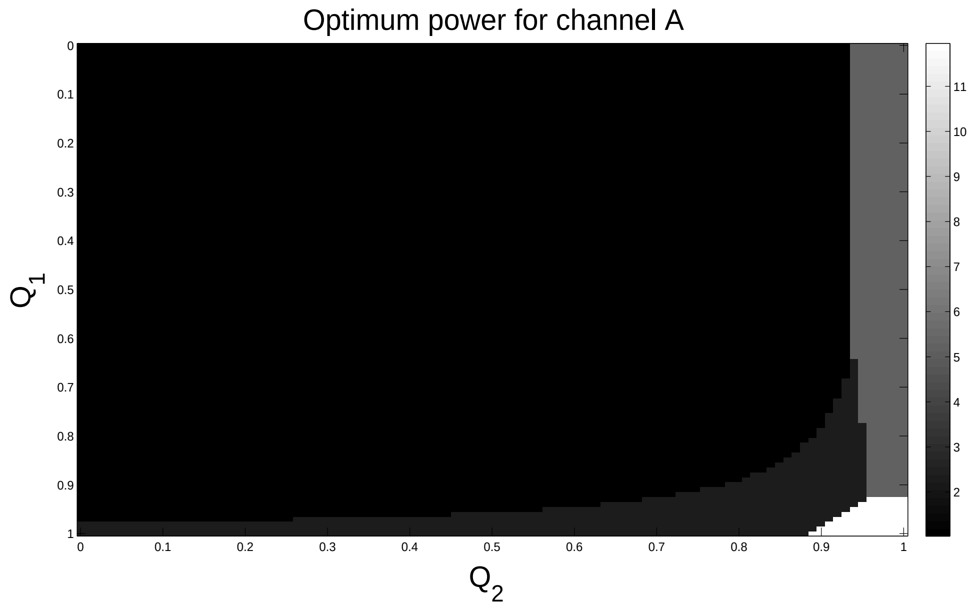

Figure 5.3: Optimum power for channel A

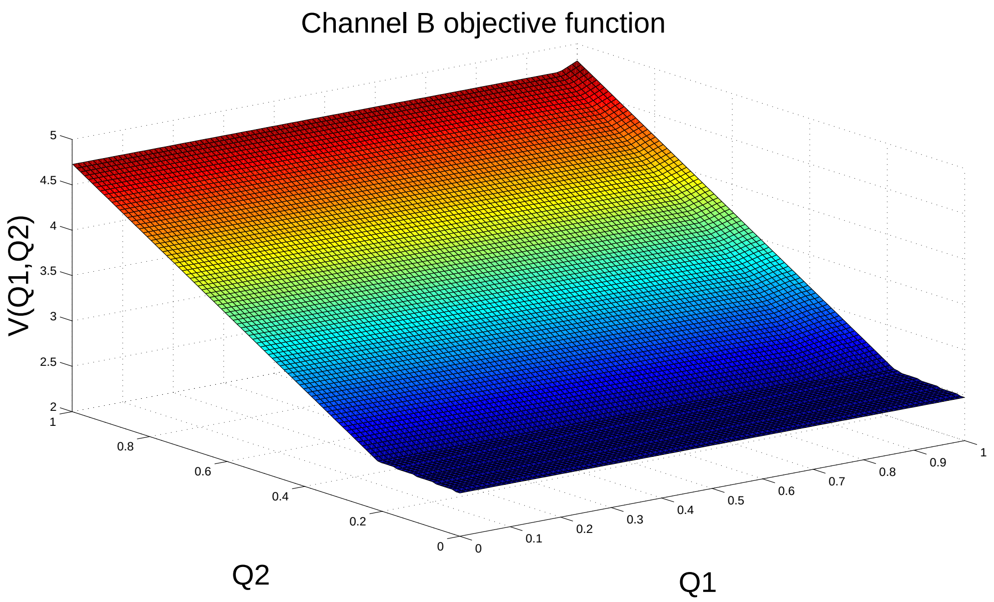

Figure 5.4: Objective function for channel B

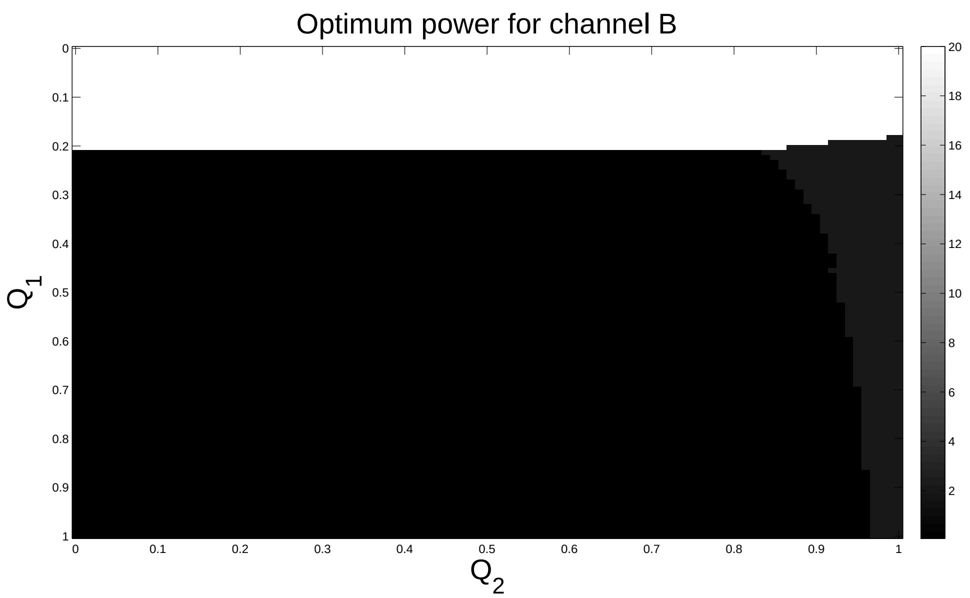

Figure 5.5: Optimum power for channel B

5.3.1 Greedy Power Control

One can look for the objective function from another perspective. Instead of looking at the future reward, the system will be a greedy one and will be interested only in the immediate reward. Hence, objective function can be obtained by putting discount factor, w, equal zero at (5.12). Recall, that greedy is not optimum. Simulation results are provided in figures 5.6, 5.7, 5.8 and 5.9

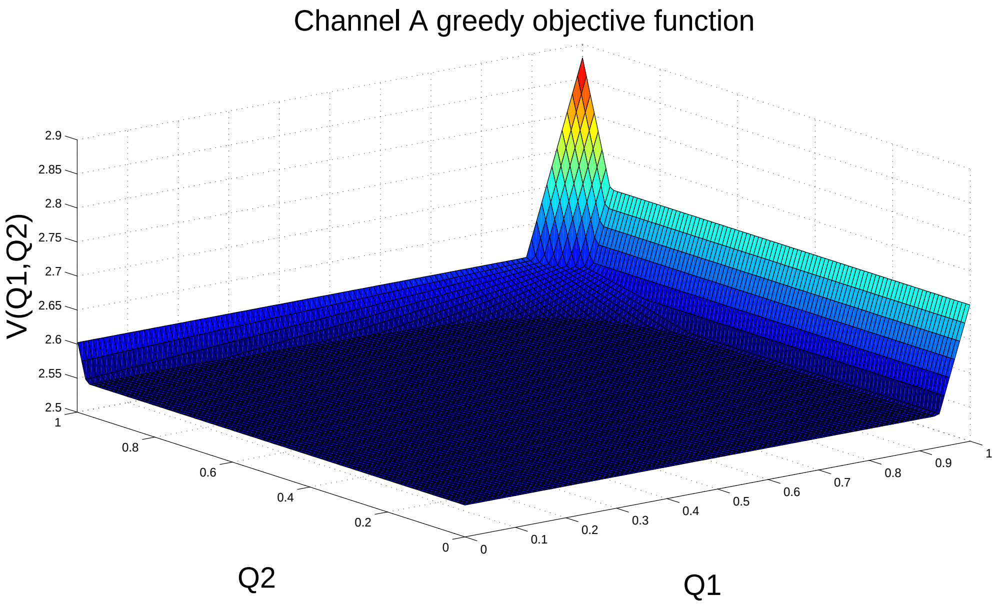

Figure 5.6: Greedy objective function for channel A

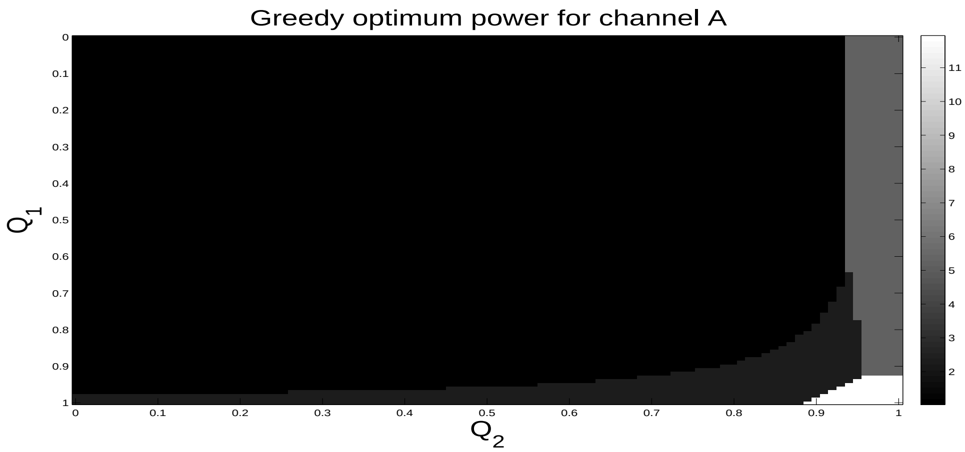

Figure 5.7: Optimum greedy power for channel A

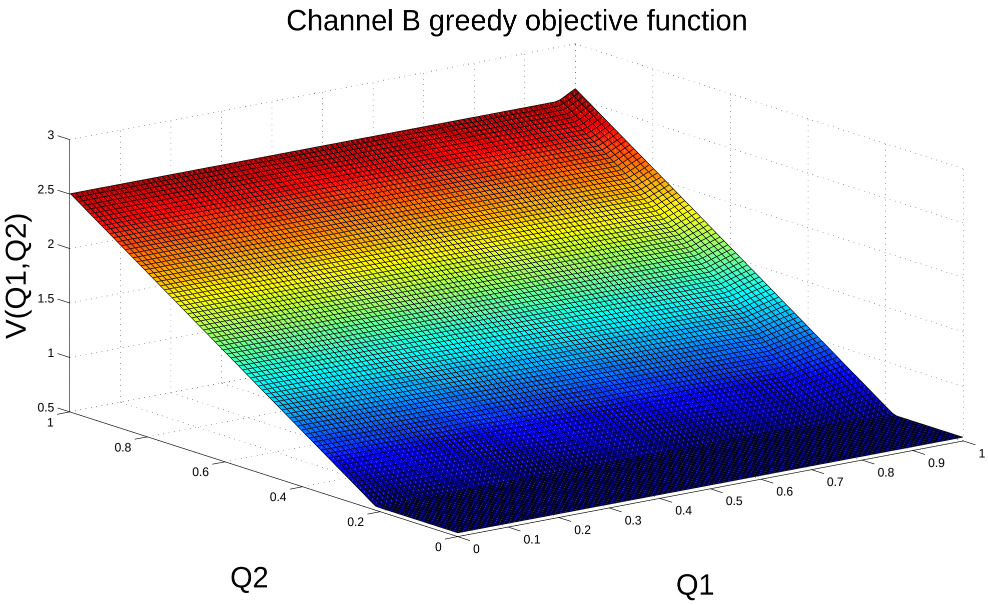

Figure 5.8: Greedy objective function for channel B

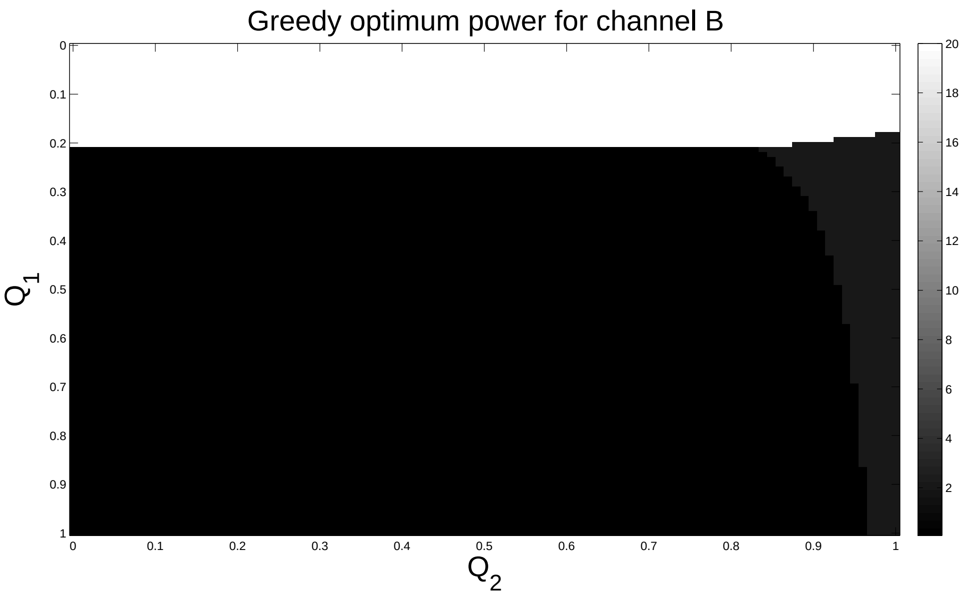

Figure 5.9: Optimum greedy power for channel B

5.3.2 Genie Aided Perspective

Another perspective, which should be an upper bound, is the genie aided. That is to say, a genie is telling the secondary user, the state of both the g_{pp} and g_{sp} channel at the next current slot. Based on the information give by genie, SU decides the optimum power that he should transmit. Hence, we have 4 values for the power corresponding to the 4 different combination of g_{pp} and g_{sp}. Recall that, there is no need to update as the genie is telling me the real next state. So in (5.3) $r(k) = r \forall k$. The optimization problem can be written as four decoupled maximization operations. The objective function can be stated as:

$$\begin{aligned} V \;=\; & \frac{1}{1-w}\Bigg[Q_{ss}\overline{Q}_{ss} \max_{P_s}\left[r(P_s \mid g_{pp}(k+1) = g_{pp}^H \;\&\; g_{sp}(k+1) = g_{sp}^L)\right] + \\ & Q_{ss}(1-\overline{Q}_{ss}) \max_{P_s}\left[r(P_s \mid g_{pp}(k+1) = g_{pp}^H \;\&\; g_{sp}(k+1) = g_{sp}^H)\right] + \\ & (1-Q_{ss})\overline{Q}_{ss} \max_{P_s}\left[r(P_s \mid g_{pp}(k+1) = g_{pp}^L \;\&\; g_{sp}(k+1) = g_{sp}^L)\right] + \\ & (1-Q_{ss})(1-\overline{Q}_{ss}) \max_{P_s}\left[r(P_s \mid g_{pp}(k+1) = g_{pp}^L \;\&\; g_{sp}(k+1) = g_{sp}^H)\right]\Bigg] \end{aligned} \tag{5.13}$$

Where Q_{ss} and $\overline{Q}_{ss}$ are the steady state probabilities for g_{pp} and g_{sp} respectively. Both can be obtained from Markov model by making next time slot state probability equals to the previous one. First term is, for instance, $r(P_s \mid g_{pp}(k+1) = g_{pp}^H \;\&\; g_{sp}(k+1) = g_{sp}^L) = (1-\alpha)Rs + \alpha r_p \gamma_1$. Recall that $\gamma_1 = P(ACK(k+1) \mid g_{pp}(k+1) = g_{pp}^H \;\&\; g_{sp}(k+1) = g_{sp}^H)$.

5.3.3 Causal System perspective

One last perspective to compare with is the causal system. If the genie is informing the secondary user the real channel state of the previous time slot **(not the next time slot)**. Hence, the objective function has to take into consideration the transition from the current state in the next time slot. As a result we have also four decoupled maximization problems and objective function is written as follows:

$$\begin{aligned} V = & \frac{1}{1-w}\Big[Q_{ss}\overline{Q}_{ss}\max_{P_s}\Big[(1-\alpha)Rs + \alpha r_p P(ACK(k+1) \mid g_{pp}(k) = g_{pp}^H \;\&\; g_{sp}(k) = g_{sp}^L)\Big] + \\ & Q_{ss}(1-\overline{Q}_{ss})\max_{P_s}\Big[(1-\alpha)Rs + \alpha r_p P(ACK(k+1) \mid g_{pp}(k) = g_{pp}^H \;\&\; g_{sp}(k) = g_{sp}^H)\Big] + \\ & (1-Q_{ss})\overline{Q}_{ss}\max_{P_s}\Big[(1-\alpha)Rs + \alpha r_p P(ACK(k+1) \mid g_{pp}(k) = g_{pp}^L \;\&\; g_{sp}(k) = g_{sp}^L)\Big] + \\ & (1-Q_{ss})(1-\overline{Q}_{ss})\max_{P_s}\Big[(1-\alpha)Rs + \alpha r_p P(ACK(k+1) \mid g_{pp}(k) = g_{pp}^L \;\&\; g_{sp}(k) = g_{sp}^H)\Big]\Big] \end{aligned} \tag{5.14}$$

The transition in the next time slot can be shown, for instance, in the following term

$$P(ACK(k+1) \mid g_{pp}(k) = g_{pp}^H \;\&\; g_{sp}(k) = g_{sp}^L) = P_{HH}\overline{P_{LL}}\gamma_1 + P_{HH}\overline{P_{LH}}\gamma_2 + P_{HL}\overline{P_{LL}}\gamma_3 + P_{HL}\overline{P_{LH}}\gamma_4 \tag{5.15}$$

Figure 5.10 proposes a comparison between the rates achieved by the genie aided scenario and the causal one for channel A. It is clear and expected that the genie aided scenario is an upper bound for the performance.

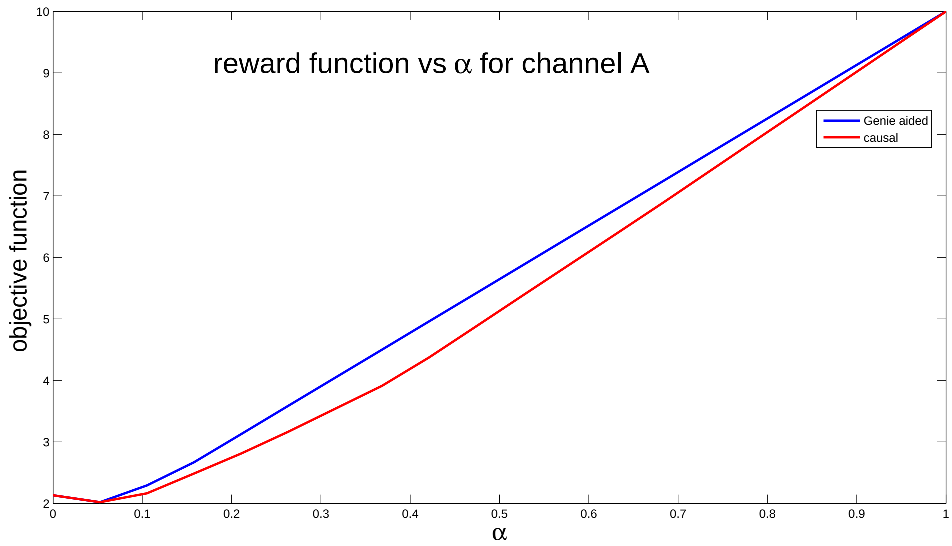

Figure 5.10: Comparison between genie aided and causal scenario

Chapter 6

Conclusion and Future Work

In this thesis we developed a novel scheme for jointly secondary user transmission time and power. Assuming the worst case uncertainty about primary activity, we find that the maximum gap between the upper and lower bounds on the secondary link ergodic capacity is always less than or equal to 1 bit per channel use. The maximum gap is generally a function of both the primary traffic parameters, and the sensing and transmission time parameters of the secondary link. In an expanded version of this work, we show that this result holds for general on/off distributions and not only for the exponential case. Future work may address the exact evaluation of $h(s)$ given the considered renewal model for primary activity. Another suggested future work is to look for an achievability scheme for this capacity.

Next, we have investigated the problem of specifying transmission power and duration in an underlay unslotted cognitive radio network, where the primary transmission duration follows an exponential distribution. We used an upper bound for the secondary throughput, and obtained, numerically, the optimal secondary transmission power and duration that maximize a weighted sum of the primary and secondary throughputs. Note that, at the particular values obtained, the solutions obtained from our optimization problem, are the same that would be obtained from a constrained optimization problem where one seeks to maximize the secondary throughput while constraining the primary throughput to be above a certain value. Our results also showed that an increase in the overall weighted throughput can be obtained by allowing the secondary to transmit even when the channel is found to be busy. We extended our formulation to the soft sensing case where the decision of the secondary transmission power and duration depends on the quantized value of the sensing metric, rather than on the binary decision of whether the channel is free or not. However, our preliminary results show that the gain of using this scheme, and for the range of parameters we have simulated, are minimal.

Finally, we shows another comparable scheme to what we introduced before which is exploiting the primary feedback message. This scenario mitigates the disadvantage of conventional listen before talk schemes. Problem is formulating as maximization for the expected future reward. Where the immediate reward is the weighted sum throughput. Results are proposed for optimum power and optimal objective function in different secondary user situations.

Appendix A

Outage Probability

$$\begin{aligned} P_\circ(p) &= \Pr\left\{ r_\circ > \log\left(1 + \frac{a g_{\mathrm{pp}}}{b g_{\mathrm{sp}} + 1}\right)\right\} \\ &= \Pr\left\{ a g_{\mathrm{pp}} - b c g_{\mathrm{sp}} < c \right\} \end{aligned} \tag{A.1}$$

where $a = P_{\mathrm{p}}/\sigma_{\mathrm{p}}^2$, $b = p/\sigma_{\mathrm{p}}^2$, and $c = \exp(r_\circ) - 1$. Assuming that g_{pp} and g_{sp} are independent and exponentially distributed with means $\overline{g}_{\mathrm{pp}}$ and $\overline{g}_{\mathrm{sp}}$, the outage probability becomes

$$\begin{aligned} P_\circ(p) &= \int_0^\infty \int_0^{\frac{c}{a}(1+b g_{\mathrm{sp}})} \frac{1}{\overline{g}_{\mathrm{pp}}} \exp(-\frac{g_{\mathrm{pp}}}{\overline{g}_{\mathrm{pp}}}) \times \frac{1}{\overline{g}_{\mathrm{sp}}} \exp(-\frac{g_{\mathrm{sp}}}{\overline{g}_{\mathrm{sp}}}) d g_{\mathrm{pp}} d g_{\mathrm{sp}} \\ &= 1 - \frac{P_{\mathrm{p}} \overline{g}_{\mathrm{pp}}}{P_{\mathrm{p}} \overline{g}_{\mathrm{pp}} + p c \overline{g}_{\mathrm{sp}}} \exp\left(-\frac{c \sigma_{\mathrm{p}}^2}{P_{\mathrm{p}} \overline{g}_{\mathrm{pp}}}\right) \end{aligned} \tag{A.2}$$

Appendix B

Secondary User Ergodic Capacity

Assuming an exponential distribution for g_{ss} with mean $\overline{g}_{\mathrm{ss}}$, (4.2) becomes

$$C_{\circ}(p) = \int_{0}^{\infty} \log\left(1 + \frac{p g_{\mathrm{ss}}}{\sigma_{\mathrm{s}}^{2}}\right) \frac{1}{\overline{g}_{\mathrm{ss}}} \exp\left(-\frac{g_{\mathrm{ss}}}{\overline{g}_{\mathrm{ss}}}\right) dg_{\mathrm{ss}} \tag{B.1}$$

Defining $\Psi(x) = \int_{x}^{\infty} \exp(-\mu)/\mu \; d\mu$, it is straightforward to show that

$$C_{\circ}(p) = \exp\left(\frac{\sigma_{\mathrm{s}}^{2}}{p\overline{g}_{\mathrm{ss}}}\right) \Psi\left(\frac{\sigma_{\mathrm{s}}^{2}}{p\overline{g}_{\mathrm{ss}}}\right) \tag{B.2}$$

Assuming that g_{ss} and g_{ps} are independent and have means $\overline{g}_{\mathrm{ss}}$ and $\overline{g}_{\mathrm{ps}}$, respectively, (4.3) can be expressed as

$$\begin{aligned} C_{1}(p) &= \int_{0}^{\infty}\int_{0}^{\infty} \log\left(1 + \frac{p g_{\mathrm{ss}}}{P_{\mathrm{p}} g_{\mathrm{ps}} + \sigma_{\mathrm{s}}^{2}}\right)) \times \\ &\quad \frac{1}{\overline{g}_{\mathrm{ss}}} \exp(-\frac{g_{\mathrm{ss}}}{\overline{g}_{\mathrm{ss}}}) \frac{1}{\overline{g}_{\mathrm{ps}}} \exp(\frac{g_{\mathrm{ps}}}{\overline{g}_{\mathrm{ps}}}) dg_{\mathrm{ss}} dg_{\mathrm{ps}} \\ &= C_{1\mathrm{a}} - C_{1\mathrm{b}} \end{aligned} \tag{B.3}$$

where

$$\begin{aligned} C_{1\mathrm{a}} &= \int_0^\infty \int_0^\infty \log\left(1 + \frac{P_\mathrm{p}}{\sigma_\mathrm{s}^2} g_\mathrm{ps} + \frac{p}{\sigma_\mathrm{s}^2} g_\mathrm{ss}\right) \times \\ & \frac{1}{\overline{g}_\mathrm{ss}} \exp(-\frac{g_\mathrm{ss}}{\overline{g}_\mathrm{ss}}) \frac{1}{\overline{g}_\mathrm{ps}} \exp(\frac{g_\mathrm{ps}}{\overline{g}_\mathrm{ps}}) dg_\mathrm{ss} dg_\mathrm{ps} \end{aligned} \tag{B.4}$$

$$\begin{aligned} C_{1\mathrm{b}} &= \int_0^\infty \log\left(1 + \frac{P_\mathrm{p} g_\mathrm{ps}}{\sigma_\mathrm{s}^2}\right) \frac{1}{\overline{g}_\mathrm{ps}} \exp\left(-\frac{g_\mathrm{ps}}{\overline{g}_\mathrm{ps}}\right) dg_\mathrm{ps} \\ &= \exp\left(\frac{\sigma_\mathrm{s}^2}{P_\mathrm{p}\overline{g}_\mathrm{ps}}\right) \Psi\left(\frac{\sigma_\mathrm{s}^2}{P_\mathrm{p}\overline{g}_\mathrm{ps}}\right) \end{aligned} \tag{B.5}$$

We find $C_{1\mathrm{a}}$ by rewriting it as

$$C_{1\mathrm{a}} = \int_0^\infty \log(1+z) f(z) \, dz \tag{B.6}$$

where $z = \frac{P_\mathrm{p}}{\sigma_\mathrm{s}^2} g_\mathrm{ps} + \frac{p}{\sigma_\mathrm{s}^2} g_\mathrm{ss}$ and $f(z)$ is the pdf of z. If $x = \frac{P_\mathrm{p}}{\sigma_\mathrm{s}^2} g_\mathrm{ps}$ and $y = \frac{p}{\sigma_\mathrm{s}^2} g_\mathrm{ss}$, then x and y are independent and have the exponential distributions

$$f_\mathrm{X}(x) = \frac{\sigma_\mathrm{s}^2}{P_\mathrm{p}\overline{g}_\mathrm{ps}} \exp\left(-\frac{\sigma_\mathrm{s}^2 x}{P_\mathrm{p}\overline{g}_\mathrm{ps}}\right), \, x \geq 0$$

and

$$f_\mathrm{Y}(y) = \frac{\sigma_\mathrm{s}^2}{p\overline{g}_\mathrm{ss}} \exp\left(-\frac{\sigma_\mathrm{s}^2 y}{p\overline{g}_\mathrm{ss}}\right), \, y \geq 0$$

respectively. The pdf $f(z)$ is the convolution of $f_\mathrm{X}(x)$ and $f_\mathrm{Y}(y)$:

$$\begin{aligned} f(z) &= \int_0^z f_\mathrm{X}(x) f_\mathrm{Y}(z-x) \, dx \\ &= \sigma_\mathrm{s}^2 \frac{\exp\left(-\frac{\sigma_\mathrm{s}^2 z}{p\overline{g}_\mathrm{ss}}\right) - \exp\left(-\frac{\sigma_\mathrm{s}^2 z}{P_\mathrm{p}\overline{g}_\mathrm{ps}}\right)}{p\overline{g}_\mathrm{ss} - P_\mathrm{p}\overline{g}_\mathrm{ps}} \end{aligned} \tag{B.7}$$

Note that in the case $p\overline{g}_{\mathrm{ss}} = P_{\mathrm{p}}\overline{g}_{\mathrm{ps}} = w$, we can use L'Hôpital's rule to get

$$f(z) = \frac{\sigma_{\mathrm{s}}^4 z}{w^2} \exp\left(-\frac{\sigma_{\mathrm{s}}^2 z}{w}\right) \tag{B.8}$$

It can then be shown that when $p\overline{g}_{\mathrm{ss}} \neq P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}$

$$\begin{aligned} C_{1\mathrm{a}} = \frac{1}{p\overline{g}_{\mathrm{ss}} - P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}} \Big[& p\overline{g}_{\mathrm{ss}} \exp\left(\frac{\sigma_{\mathrm{s}}^2}{p\overline{g}_{\mathrm{ss}}}\right) \Psi\left(\frac{\sigma_{\mathrm{s}}^2}{p\overline{g}_{\mathrm{ss}}}\right) - \\ & P_{\mathrm{p}}\overline{g}_{\mathrm{ps}} \exp\left(\frac{\sigma_{\mathrm{s}}^2}{P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}}\right) \Psi\left(\frac{\sigma_{\mathrm{s}}^2}{P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}}\right) \Big] \end{aligned} \tag{B.9}$$

Therefore,

$$\begin{aligned} C_1(p) = \frac{p\overline{g}_{\mathrm{ss}}}{p\overline{g}_{\mathrm{ss}} - P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}} \Big[& \exp\left(\frac{\sigma_{\mathrm{s}}^2}{p\overline{g}_{\mathrm{ss}}}\right) \Psi\left(\frac{\sigma_{\mathrm{s}}^2}{p\overline{g}_{\mathrm{ss}}}\right) - \\ & \exp\left(\frac{\sigma_{\mathrm{s}}^2}{P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}}\right) \Psi\left(\frac{\sigma_{\mathrm{s}}^2}{P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}}\right) \Big] \end{aligned} \tag{B.10}$$

In the case $p\overline{g}_{\mathrm{ss}} = P_{\mathrm{p}}\overline{g}_{\mathrm{ps}} = w$,

$$C_{1\mathrm{a}} = \int_0^\infty \frac{\sigma_{\mathrm{s}}^4 z}{w^2} \exp\left(-\frac{\sigma_{\mathrm{s}}^2 z}{w}\right) \log(1+z)\, dz$$

Integrating by parts we obtain

$$C_{1\mathrm{a}} = 1 + \left(1 - \frac{\sigma_{\mathrm{s}}^2}{w}\right) \exp\left(\frac{\sigma_{\mathrm{s}}^2}{w}\right) \Psi\left(\frac{\sigma_{\mathrm{s}}^2}{w}\right)$$

and, hence, when $p\overline{g}_{\mathrm{ss}} = P_{\mathrm{p}}\overline{g}_{\mathrm{ps}}$

$$C_1(p) = 1 - \frac{\sigma_{\mathrm{s}}^2}{p\overline{g}_{\mathrm{ss}}} \exp\left(\frac{\sigma_{\mathrm{s}}^2}{p\overline{g}_{\mathrm{ss}}}\right) \Psi\left(\frac{\sigma_{\mathrm{s}}^2}{p\overline{g}_{\mathrm{ss}}}\right) \tag{B.11}$$

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